Using MILP (Mixed Integer Linear Programming) for RF Bandwidth Optimization

A Technical Paper prepared for SCTE•ISBE by

Tom Holloran
Principal Operations Research Scientist
Charter Communications
14810 Grasslands Dr, Englewood, CO 80112
720.518.2296
thomas.j.holloran@charter.com

Table of Contents

Title **Page Number**

List of Figures

Introduction

While DOCSIS software advances and video analog to digital conversions have dramatically increased the amount of video and data that can be delivered across a MHz of spectrum, annual IP data growth rates of 40-60% have made a 6 MHz EIA slot of spectrum an increasingly valuable commodity. Managing MSO product delivery on a fixed amount of RF spectrum has also become very challenging -- to the point that each slot of spectrum requires management like any other resource in the MSO supply chain.

This paper describes a set of algorithms and a supply chain process that can be used to identify the absolute minimum number of EIA slots that are required to meet the demand for any set of existing and future linear and switched video, digital audio, and IP data. Additionally, the algorithms can identify the minimum number of changes that are required to move content from existing spectrum locations into the identified minimum EIA slot allocation, and can maximize the placement preferences for content within defined ranges of spectrum.

The optimization algorithms of this paper are based on integer linear programming. However, they are implemented using a data-driven approach where tables containing engineering rules and location preferences drive all mathematical model generation. The underlying optimization system allows the modeler to treat content and EIA slots as supply chain commodities. Detailed changes to the optimization model can be made without in-depth knowledge or training in mathematical programming optimization techniques.

The algorithms and process contained in this paper are not limited in any way by the hardware and software required for implementation. Most mathematical optimization software packages that have a generalized integer programming capability can be used to implement the algorithms. A variety of software packages exist that can be used to visualize optimized RF Channel Maps. And a variety of software programming languages will facilitate the required functions of pattern generation and solution fitting.

System and Process Description

1. Business Requirements

The business requirements for RF bandwidth optimization include the following:

- Minimization of the number of 6 MHz EIA slots being used for voice and data in each RF Channel Map
- Assignment of voice and data individual content to the preferred frequency locations
- Assignment of voice and data content to frequency ranges to those defined in standardized RF channel map templates
- Minimization of content movement from existing locations in order to accommodate minimum slot utilization and location into standardized/preferred frequency ranges
- Maximization of quality and capacity by adherance to all engineering constraints
- Implementation of a system and process that will reduce the time required to design RF channel map reconfigurations, that can be used by a large number of users, that does not require knowledge of advanced mathematics to use it, and enables use for both tactical (short-term) and strategic (longer term) planning.

1.1. Minimization of the number of 6 MHz EIA slots

Minimizing the number of EIA channels, total frequency, or bandwidth required is the primary goal of spectral bandwidth optimization. By minimizing the bandwidth in use, EIA channels are opened up to accommodate video and/or data growth. Identifying the minimum number of EIA slots determines when plants need to be upgraded, how upgrades can be avoided, what/when architectural changes affecting capacity need to be made, etc. The EIA slot is the highest value asset assumed in optimizing bandwidth.

1.2. Assignment of voice and data individual content to preferred frequency locations

Given a specific minimum number of EIA slots required, a secondary business requirement is placing voice and data content into preferred frequencies, and avoiding certain reserved frequencies. Preferred locations may be designated for quality reasons, or they may just be preferences to standardize locations across plants for ease of maintenance/continuity.

1.3. Assignment of voice and data content according to standardized templates

A business requirement to move toward standardizing the locations of certain types of content makes broad engineering changes or capacity upgrades easier to implement. Standardized template implementation in bandwidth optimization is handled as a preference, a secondary objective to minimizing the amount of bandwidth that a set of content consumes. However, the business requirement for standardization does allow for setting preference costs high enough that they become more important than other location preferences, or even total EIA slot bandwidth.

1.4. Minimization of moves/changes required to use only the minimum number of EIA slots

Another business requirement in optimizing bandwidth is minimizing the amount of reconfiguration and content movement that is necessary to use only the minimum number of EIA slots. There are alternative optimum (alternative configurations that use the minimum number of EIA slots), so this business requirement specifically addresses identifying the set of content assignments that minimize bandwidth use but also result in the minimum disruption to the existing RF channel map.

1.5. Adherance to engineering constraints

Adherance to engineering constraints may be an obvious business requirement. But it is important that they are not overlooked as they definitely effect any bandwidth optimization solution. Linear video content may have attached contractual constraints. Engineering may prefer contiguous blocks of DOCSIS be a certain width or minimum width based on capacity testing. DOCSIS 3.1 OFDM blocks must be contiguous to take advantage of their inherent design. Operations capacity engineers may need open EIA slots in a location that is planned to be expanded in the near future. Video engineers may require SDV and VOD pools to be adjacent to facilitate future changes. A bandwidth optimization system needs to be developed to accommodate these types of engineering constraints, but to the extent possible, must also anticipate that other constraints will be required as architectures and softtware change over time.

1.6. System and Process Design

Business requirements for system and process design can be summarized in the following bullet points for design goals:

- System and process design should reduce the time required to identify needed bandwidth reconfigurations
- System and process design should be capable of accommodating a large number of users
- System and process design should not require advanced math knowledge to operate
- System and process design should accommodate both tactical (short-term) and strategic (longer-term) planning
    - Use by individual head end engineers on individual channel maps, accommodating local requirements for reserved frequencies, use of rolloff frequencies, etc.
    - Use by strategic corporate planners to identify minimum bandwidth upgrades required system wide by certain business planning scenarios, to produce 5-year budgeting projections on service group splits, plant upgrades, etc, to estimate the impact of traffic growth projections and architectural design changes, and other corporate-wide business initiatives

2. Technical Approach

The RF bandwidth optimization technical approach is based on the following:

- Use of optimization vs. heuristics
- Flexible and streamlined processes through data-driven design
- Cost metrics derived by Finance corporate
- Development of a data repository
- Ongoing ROI tracking and management

2.1. Optimization versus Heuristic Approach

A heuristic approach uses best practices or approximation techniques to identify a good solution from among the set of all alternatives. An optimization approach evaluates all alternatives and identifies the absolute best obtainable solution from among that full set of alternatives. Simply evaluating all possible alternatives (explicity enumeration) is the most straightforward method of optimization. There are also mathematical optimization techniques like linear programming which can implicitly (versus explicitly) evaluate all alternatives through the use of convergence algorithms. They are able to identify the absolute best obtainable alternative without having to evaluate all alternatives individually.

The optimization approach was chosen over a heuristic approach due to the high value placed on each 6MHz slot of spectrum. We want to know that the absolute minimum bandwidth requirement has been identified. The mixed integer linear programming optimization approach was chosen due to the sheer number of alternatives that must be evaluated to guarantee the minimum bandwidth requirement has been found. If there are 50 patterns (unique sets of content that can be placed on an EIA slot) and 116 EIA slots (750 MHz plant, for example), then the total number of pattern to EIA slot combinations is 50*116 or 5,800. Each pattern combination is either chosen or not chosen, resulting in a combined set of 2^{5800} sets of assignments that must be evaluated to determine optimality.

The mixed integer linear programming optimization approach uses the mathematical formulation shown in Figure 1. It is based on a cutting stock formulation that has been used in manufacturing for decades. Because of its cutting stock structure, extremely large combinatorial problems can be solved very quickly. In the case of RF bandwidth optimization, the optimization algorithm generally takes less that 10 seconds on a standard-issue laptop to identify an optimal solution from among 2^{5800} possible alternatives.

The interpretation of the mathematical formulation in Figure 1 can be stated simply as:

> Minimize the number of required EIA slots while meeting as many assignment preferences as possible in the bandwidth available, and fitting as much content as possible into the bandwidth available.

Minimize:

$$\sum_{i=1}^{M}\sum_{j=1}^{N} c_{ij}x_{ij} + \sum_{i=1}^{M} e_i y_i + \sum_{k=1}^{K} f_k z_k$$

Subject to:

$$\sum_{i=1}^{M}\sum_{j=1}^{N} a_{jk}x_{ij} + z_k \geq D_k \quad \forall\, k, k = 1, \ldots, K$$

$$\sum_{j=1}^{N} x_{ij} - 1000\, y_i \leq 0 \quad \forall\, i, i = 1, \ldots, M$$

$$\sum_{j=1}^{N} x_{ij} \leq 1 \quad \forall\, i, i = 1, \ldots, M$$

where: $yi = 0$ or 1 (1 if EIA i is used, 0 if EIA i is left open)

$x_{ij} = 0$ or 1 (1 if pattern j is assigned to EIA i, 0 otherwise)

$z_k =$ the number of content type k assignments that cannot be accommodated in the channel map

$c_{ij} =$ preference cost plus the cost of changing content assignments when pattern j is assigned to EIA i,

$e_i =$ cost of using EIA slot i,

$f_k =$ cost of not being able to accommodate a unit of demand for content assignment type k,

$a_{jk} =$ number of content assignments of type k in pattern j,

$D_k =$ number of content assignments of type k required in the RF Channel Map

$N =$ number of individual patterns (sets of content assignments that fill an EIA slot) that can be used

$M =$ number of EIA's available for possible use

$K =$ total number of content assignment types (e.g. HD, SD, DA, SDV, VOD, DOCSIS, RESERVED)

Figure 1 - RF Spectral Bandwidth Mathematical Formulation

2.2. Data Driven Approach

An approach that is data driven simply implies that by setting up tables of differing size and content, a customized mathematical model can be constructed and solved. The contents of the tables drive varying sized of RF channel maps as well as varying sets of engineering constraints and operational preferences. Data driven also implies that a model that is constructed will also always solve. No infeasible solutions are possible. Identification of a shortfall in bandwidth or an inability to accommodate certain content due to lack of allowable assignment patterns is merely identified as part of the optimization results.

Figure 2 contains part of an RF channel map. Part of adopting a data driven approach means that total content bandwidth requirements, total number of EIA slots in the channel map, total moves required to change to a minimum set of EIAs, etc. can and will all be derived from the read of the channel map.

Hub CLLI	ChannelMap	EIAChannel	CurrentAllocation	Bandwidth	Bandwidth Type
TRCYMI21713	Traverse City Region, MI (tc_om01m)	2	HBO 2 HD West	9.5	HD4
TRCYMI21713	Traverse City Region, MI (tc_om01m)	2	HBO Family HD West	9.5	HD4
TRCYMI21713	Traverse City Region, MI (tc_om01m)	2	HBO HD West	9.5	HD4
TRCYMI21713	Traverse City Region, MI (tc_om01m)	2	HBO Signature HD West	9.5	HD4
TRCYMI21713	Traverse City Region, MI (tc_om01m)	11	Charter Mainstreet	2.75	SD
TRCYMI21713	Traverse City Region, MI (tc_om01m)	11	Local Access	2.75	SD
TRCYMI21713	Traverse City Region, MI (tc_om01m)	11	PBS	2.75	SD
TRCYMI21713	Traverse City Region, MI (tc_om01m)	11	PEG Access	2.75	SD
TRCYMI21713	Traverse City Region, MI (tc_om01m)	11	Public Access	2.75	SD
TRCYMI21713	Traverse City Region, MI (tc_om01m)	15	Independent TV Station HD	19.4	HD2
TRCYMI21713	Traverse City Region, MI (tc_om01m)	15	PBS HD	19.4	HD2
TRCYMI21713	Traverse City Region, MI (tc_om01m)	16	CLI analog carrier	38.8	RESERVED
TRCYMI21713	Traverse City Region, MI (tc_om01m)	17	Comedy Central HD East	12.75	HD3
TRCYMI21713	Traverse City Region, MI (tc_om01m)	17	NBCSN HD (NBC Sports Network)	12.75	HD3
TRCYMI21713	Traverse City Region, MI (tc_om01m)	17	TBS HD East (Turner Broadcasting System)	12.75	HD3
TRCYMI21713	Traverse City Region, MI (tc_om01m)	18	MC 70s	0.4	MC
TRCYMI21713	Traverse City Region, MI (tc_om01m)	18	MC 80s	0.4	MC
TRCYMI21713	Traverse City Region, MI (tc_om01m)	18	MC 90s	0.4	MC
TRCYMI21713	Traverse City Region, MI (tc_om01m)	61	Analog Pilot	38.8	ANALOG
TRCYMI21713	Traverse City Region, MI (tc_om01m)	62	VOD Channel	38.8	VOD
TRCYMI21713	Traverse City Region, MI (tc_om01m)	63	VOD Channel	38.8	VOD
TRCYMI21713	Traverse City Region, MI (tc_om01m)	74	AVN	38.8	RESERVED
TRCYMI21713	Traverse City Region, MI (tc_om01m)	75	AVN	38.8	RESERVED
TRCYMI21713	Traverse City Region, MI (tc_om01m)	76	OPEN	38.8	OPEN
TRCYMI21713	Traverse City Region, MI (tc_om01m)	77	Zodiac	38.8	RESERVED
TRCYMI21713	Traverse City Region, MI (tc_om01m)	79	SDV Channel	38.8	SDV
TRCYMI21713	Traverse City Region, MI (tc_om01m)	80	SDV Channel	38.8	SDV
TRCYMI21713	Traverse City Region, MI (tc_om01m)	87	DOCSIS Channel	38.8	DOCSIS3.0
TRCYMI21713	Traverse City Region, MI (tc_om01m)	88	DOCSIS Channel	38.8	DOCSIS3.0
TRCYMI21713	Traverse City Region, MI (tc_om01m)	89	DOCSIS Channel	38.8	DOCSIS3.0
TRCYMI21713	Traverse City Region, MI (tc_om01m)	90	DOCSIS Channel	38.8	DOCSIS3.0
TRCYMI21713	Traverse City Region, MI (tc_om01m)	91	DOCSIS Channel	38.8	DOCSIS3.0
TRCYMI21713	Traverse City Region, MI (tc_om01m)	92	DOCSIS Channel	38.8	DOCSIS3.0
TRCYMI21713	Traverse City Region, MI (tc_om01m)	93	DOCSIS Channel	38.8	DOCSIS3.0
TRCYMI21713	Traverse City Region, MI (tc_om01m)	94	DOCSIS Channel	38.8	DOCSIS3.0

Figure 2 - RF Channel Map Contents

2.3. Finance-infused Cost Metrics

A technical approach that uses cost metrics derived from corporate-backed financial reporting is very important to identifying appropriate tradeoffs in bandwidth design. It is also important to accurately track improvements ongoing and justify the use of an optimization approach. Finance metrics leveraged should include CAPEX, OPEX, EBITDA, etc. The key metric in RF bandwidth optimization is the cost of a 6 MHz EIA slot. Reconfiguration FTE and hardware/software costs are also important. Derivation of future costs required for longer-term strategic planning should also be based on hardened corporate financial projections whenever possible.

2.4. Data Repository Creation

The technical approach to RF bandwidth optimization should incorporate creation of a data repository. Automation of data creation/read and configuration changes/write is ideal, but creation and maintence of a data repository of RF channel map configurations and cost in any form is an absolute requirement for bandwidth optimization to be successful.

Figure 3 shows a system overview containing a data repository and its interfaces.

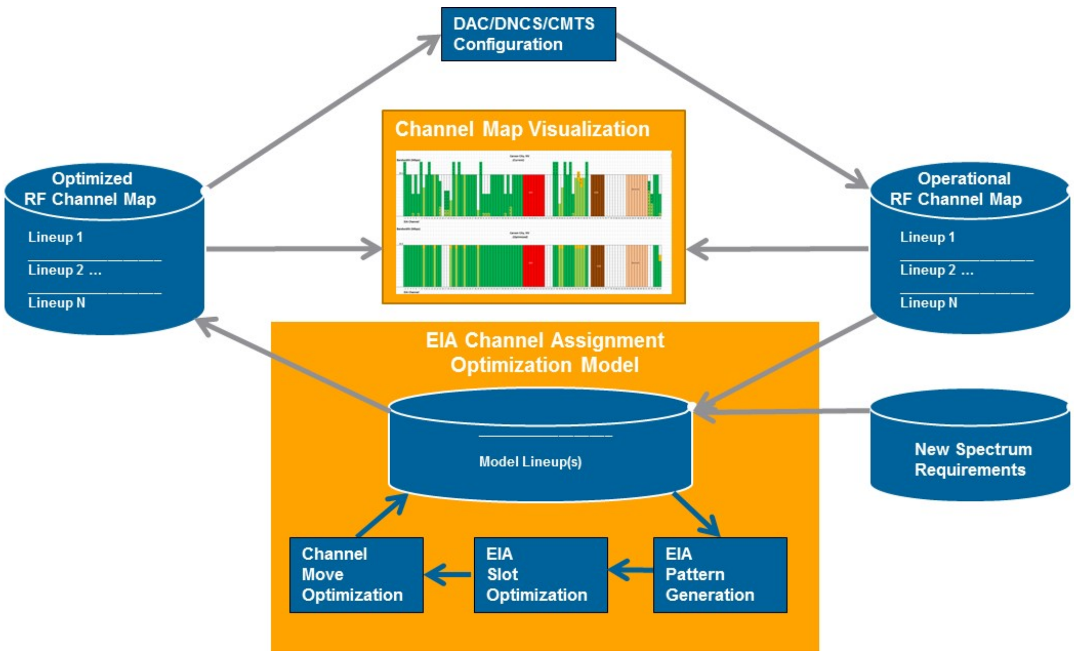

Figure 3 - System Configuration Overview

2.5. ROI Tracking

ROI tracking of improvements is a key component to justifying the investment in optimization. It is also important to identify potential changes and improvements that can be gained based on costs incurred and/or improved.

Successul creation of an RF bandwidth optimization system is best accomplished using agile development. Figure 4 shows the closed-loop agile analytics process adopted as part of the RF bandwidth optimization technical approach.

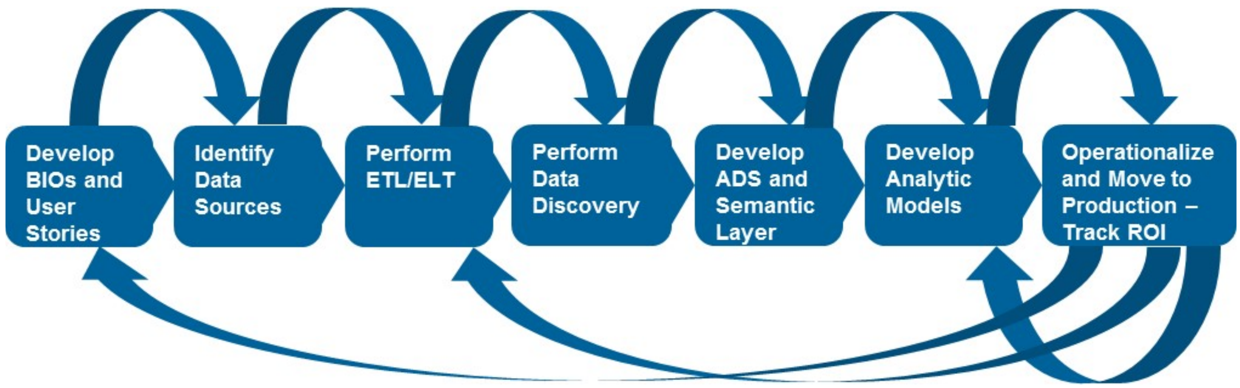

Figure 4 - Analytics Agile Development Lifecycle

3. RF Bandwidth Optimization System

Major components of the RF bandwidth optimization system consist of an optimization modeling platform, a set of driving control tables, bandwidth assignment visualization tools, and the method of model data / model execution management.

3.1. Modeling Platform

The modeling platform consists of a mathematical model generator, and optimizer, and a solution formatter. The model generator and solution formatter can be created using a wide variety of programming languages and/or data blending tools. The optimization module requires commercial mathematical programming software to solve the larger spectral bandwdith problems. A 750 MHz plant can easily produce 3000+ alternative assignments or 2^{3000} combinations to be evaluated in order to identiy an optimal set of assignments. Fortunately commercial mathematical programming software packages are capable of solving these problems in less than 1 minute. In the RF Bandwidth Optimization system, model optimization averages only about 10 seconds.

A graphical representation of the optimization model is shown in Figure 5. In supply chain terms an inventory of EIA slots is established, EIA slots are cut into patterns representing all combinations of allowable sets of content that will fit on an EIA, EIA patterns are then cut into individual pieces of content that are matched against the demand required to accommodate a defined RF channel map. The flow is then submitted to the optimizer which determines the minimum number of EIA slots that is required to meet all content demand, and at least cost.

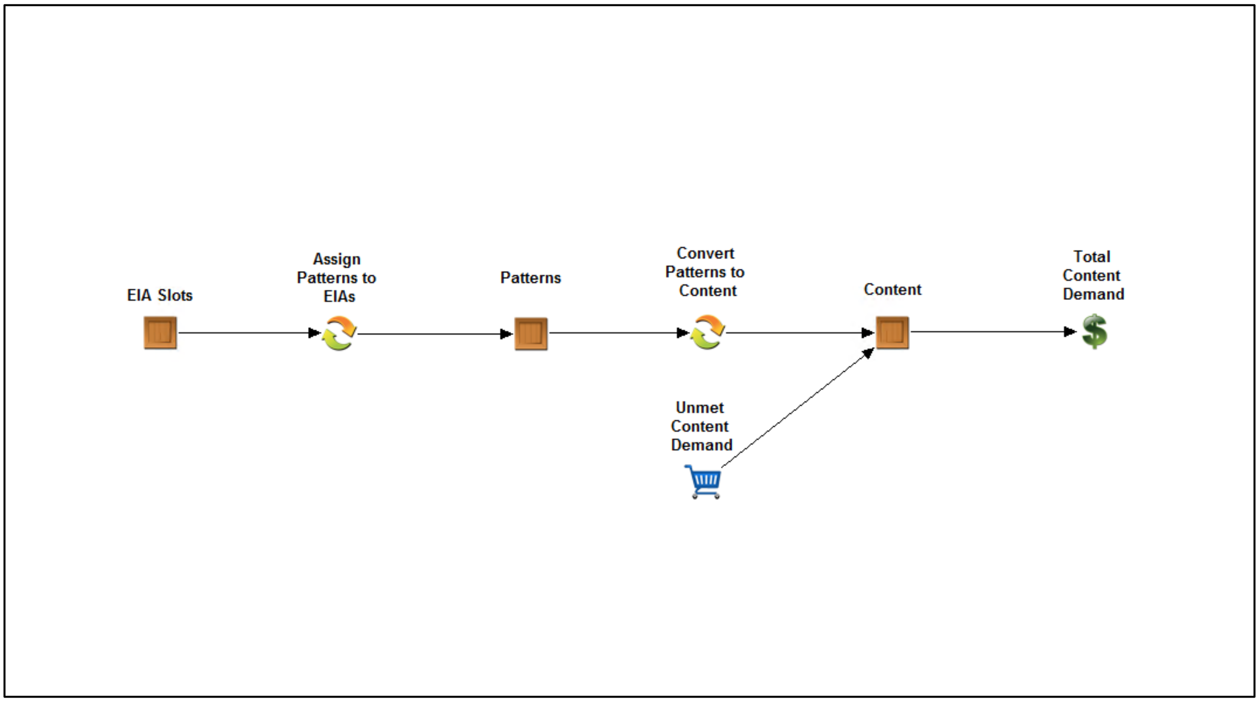

Figure 5 - Mathematical Model Visualization

3.2. Control Tables

There are six control tables that define the RF bandwidth being modeled along with all of the content demand requirement and engineering/operations preferences for assigning content to bandwidth.

ChannelMap			
ChannelMap	EIAChannel	Type	CurrentAllocation
Traverse City Region, MI 750	73	SD	In Demand PPV
Traverse City Region, MI 750	73	SD	Showtime West
Traverse City Region, MI 750	73	SD	Starz East
Traverse City Region, MI 750	73	SD	ThrillerMax East
Traverse City Region, MI 750	74	RESERVED	AVN
Traverse City Region, MI 750	75	RESERVED	AVN
Traverse City Region, MI 750	76	OPEN	OPEN
Traverse City Region, MI 750	77	RESERVED	Zodiac
Traverse City Region, MI 750	78	SD	C-Span 3
Traverse City Region, MI 750	78	SD	Discovery Channel East
Traverse City Region, MI 750	78	SD	ESPN 2
Traverse City Region, MI 750	78	SD	FX East
Traverse City Region, MI 750	78	SD	HGTV East (Home & Garden Television)
Traverse City Region, MI 750	79	SDV	SDV Channel
Traverse City Region, MI 750	80	SDV	SDV Channel
Traverse City Region, MI 750	81	SDV	SDV Channel
Traverse City Region, MI 750	82	SDV	SDV Channel
Traverse City Region, MI 750	83	SDV	SDV Channel
Traverse City Region, MI 750	84	SDV	SDV Channel
Traverse City Region, MI 750	85	SDV	SDV Channel
Traverse City Region, MI 750	86	SDV	SDV Channel
Traverse City Region, MI 750	87	DOCSIS 3.0	DOCSIS Channel
Traverse City Region, MI 750	88	DOCSIS 3.0	DOCSIS Channel
Traverse City Region, MI 750	89	DOCSIS 3.0	DOCSIS Channel
Traverse City Region, MI 750	90	DOCSIS 3.0	DOCSIS Channel
Traverse City Region, MI 750	91	DOCSIS 3.0	DOCSIS Channel
Traverse City Region, MI 750	92	DOCSIS 3.0	DOCSIS Channel
Traverse City Region, MI 750	93	DOCSIS 3.0	DOCSIS Channel
Traverse City Region, MI 750	94	DOCSIS 3.0	DOCSIS Channel
Traverse City Region, MI 750	95	HD4	Bloomberg TV HD

Figure 6 - Operational RF Channel Map

The Operational RF Channel Map identifies all of the content currently being accommodated on the plant along with its EIA slot location. Total content is identified as the base content requirement to be optimized along with the number of EIA channels (spectrum size) in the channel map to be optimized.

Figure 6 shows a sample portion of an Operational RF Channel Map – containing rows identifying plant name, EIA slot, content type, and specific content name.

Bandwidth Demand			
	Exisiting Demand	Additional Demand	Total Demand
SD	138	0	138
SD2	0	0	0
HD	0	0	0
HD2	6	0	6
HD3	31	0	31
HD4	121	0	121
DA	44	0	44
DOCSIS 3.0	24	0	24
DOCSIS 3.1	0	16	16
SDV	8	0	8
VOD	6	0	6
SDV-VOD	0	0	0
RESERVED	6	0	6
OPEN	0	0	0

Figure 7 - Bandwidth Demand Control Table

A bandwidth demand control table like that shown in Figure 7 is generated based on information in the RF channel map of Figure 6.

The bandwidth demand control table contains an additional demand column where content demand derived from the RF channel map can be modified (added to or subtracted from) to model future demand or account for variations from a base map.

Patterns										
	Contiguous	SD	SD2	HD	HD2	HD3	HD4	DA	D3.0	D3.1
Pattern01:2-0-0-0-0	1	0	0	0	2		0	0	0	0
Pattern02:1-1-0-2-0	1	2	0	0	1	1	0	0	0	0
Pattern03:1-1-0-1-8	1	1	0	0	1	1	0	8	0	0
Pattern04:1-1-0-0-13	1	0	0	0	1	1	0	13	0	0
Pattern05:1-0-2-0-0	1	0	0	0	1	0	2	0	0	0
Pattern06:1-0-1-3-0	1	3	0	0	1	0	1	0	0	0
Pattern07:1-0-1-2-8	1	2	0	0	1	0	1	8	0	0
Pattern08:1-0-1-1-14	1	1	0	0	1	0	1	14	0	0
Pattern09:1-0-1-0-19	1	0	0	0	1	0	1	19	0	0
Pattern10:1-0-0-6-0	1	6	0	0	1	0	0	0	0	0

Figure 8 - Patterns Control Table

A patterns control table like the sample shown in Figure 8 is also generated off of the RF channel map contents. Every combination of content that can be accommodated on a single EIA slot is identified as a pattern. Prior to executing the optimization, undesired patterns can be removed from the table. The patterns control table Contiguous column is used to identfy the number of an individual pattern type that must exist adjacent to each other.

Preferences	EIA002	EIA003	EIA004	EIA005	EIA006	EIA095	EIA096	EIA097	EIA098	EIA099	EIA014	EIA015	EIA016
Pattern01:2-0-0-0-0	0	0	0	0	0	0	0	0	0	0	0	0	1000
Pattern02:1-1-0-2-0	0	0	0	0	0	0	0	0	0	0	0	0	1000
Pattern03:1-1-0-1-8	0	0	0	0	0	0	0	0	0	0	0	0	1000
Pattern04:1-1-0-0-13	0	0	0	0	0	0	0	0	0	0	0	0	1000
Pattern05:1-0-2-0-0	0	0	0	0	0	0	0	0	0	0	0	0	1000
Pattern06:1-0-1-3-0	0	0	0	0	0	0	0	0	0	0	0	0	1000
Pattern07:1-0-1-2-8	0	0	0	0	0	0	0	0	0	0	0	0	1000
Pattern08:1-0-1-1-14	0	0	0	0	0	0	0	0	0	0	0	0	1000
Pattern09:1-0-1-0-19	0	0	0	0	0	0	0	0	0	0	0	0	1000
Pattern10:1-0-0-6-0	0	0	0	0	0	0	0	0	0	0	0	0	1000
Pattern81:DOCSIS3.0	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
Pattern82:DOCSIS3.1	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
Pattern83:SDV	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
Pattern84:VOD	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
Pattern85:RESERVED	1000	1000	1000	1000	1000	1000	0	1000	1000	1000	1000	1000	0
Pattern86:OPEN	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 9 - Preferences Control Table

The preferences control table is also generated from the base RF channel map of Figure 6. It contains a preference cost for assigning an individual set of content (pattern) to each EIA. The set of patterns is based on the Patterns table of Figure 8. Default preference costs are placed into the table, but can be modified as needed/desired.

This is the control table that is used to bias alternative minimum EIA optimal solutions toward preferred frequency locations for content, or toward established standardization guidelines.

Moves	EIA002	EIA003	EIA004	EIA005	EIA006	EIA095	EIA096	EIA097	EIA098	EIA099	EIA014	EIA015	EIA016
Pattern01:2-0-0-0-0	4	4	4	4	4	3	1	2	2	12	3	0	1
Pattern02:1-1-0-2-0	4	4	4	4	4	2	1	2	2	10	3	1	1
Pattern03:1-1-0-1-8	4	4	4	4	4	2	1	2	2	11	3	1	1
Pattern04:1-1-0-0-13	4	4	4	4	4	3	1	2	2	12	3	1	1
Pattern05:1-0-2-0-0	2	2	2	2	2	1	1	0	0	12	1	1	1
Pattern06:1-0-1-3-0	3	3	3	3	3	1	1	1	1	9	2	1	1
Pattern07:1-0-1-2-8	3	3	3	3	3	1	1	1	1	10	2	1	1
Pattern08:1-0-1-1-14	3	3	3	3	3	1	1	1	1	11	2	1	1
Pattern09:1-0-1-0-19	3	3	3	3	3	2	1	1	1	12	2	1	1
Pattern10:1-0-0-6-0	4	4	4	4	4	2	1	2	2	6	3	1	1

Figure 10 - Moves Control Table

A moves control table is also auto-generated based on the RF channel map initial assignments – identifying the number of content assignments that will change if a specific set of content (pattern) is assigned to an individual EIA.

This table is used to bias alternative optimal minimum EIA solutions toward those with the least impact/disruption. The weight that is placed on biasing toward move minimization is set through a single global move cost in the Levers control table of Figure 11.

Levers		
Lever	**Value**	**Description**
Objective	1	Optimization Options: 1=Minimize the number of EIA channels required, 2=Minimize the number of EIAs required, the cost of Moves/re- and the Preference costs associated with placing individual patterns on individual EIAs
EIA Channel Value	$100,000	The value of each EIA channel
Move Unit Cost	$1,000	The unit cost per move associated with moving or re-assigning a currently allocated unit from its assigned EIA channel to another EIA channel
Minimum EIA Channels	1	The minimum number of EIA channels to remain in use. A value greater than the minimum number of EIA relaxes the model objective.

Figure 11 - Levers Control Table

This Levers contol table of Figure 11 is used to set optimization modeling objectives and assign costs to EIA slots and individual content moves.

Depending on the level of detail being analyzed in the optimization, the Objective can be set to 1) simply identify the minimum number of EIA slots required to accommodate bandwidth demand or 2) to identify the minimum EIA slot solution that also maximizes content assignment preferences and minimizes changes from existing content assignment locations.

EIA Channel Value is used to indicate potential savings through minimum EIA slot use, and also to weight the relative importance of minimizing the number of EIA slots versus the importance of minimizing disruption and maximizing assignment location preferences.

By manipulating the value of the Move Unit Cost, different levels of emphasis on disruption can be defined.

By manipulating the Minimum EIA Channels value, re-assignment of content can be reduced at the cost of using more than the minimum number of EIA slots. This is specifically accomplished by defining a Minimum EIA Channel count that is greater than the minimum number of EIA slots required to accommodate the content – established in a previous run of the optimization model.

3.3. Optimized RF Channel Map

The EIA Channel Assignment Optimization Model system component of Figure 3 generates a mathematical model based on the control tables, optimizes the model using a mixed integer linear programming optimizer, and builds an Optimized RF Channel Map based on the resulting optimized solution.

The Optimized RF Channel Map contains the same content as the original Operational RF Channel Map of Figure 6 (rows identifying plant name, EIA slot, content type, and specific content name) – but reflects the optimized assignments of content to EIA – assignments that minimize EIA slot utilization, maximize preferred location of content, etc.

3.4. Results Visualization

A standard summary of optimization model results is shown in Figure 12. It shows that through optimization, an additional 13 EIA slots of spectrum can be freed up. The minimum number of moves required in order to minimize EIA slot use is 35. If freeing an EIA slot is valued at $100K then the total resulting improvement is shown as $1.3 million, or $1.265 million if each content move reduces the benefit by $1K. The Optimal Patterns table identifies the variouis EIA slot configurations that would be used to produce the minimum EIA slot configuration. For example, 28 EIA slots would be comprised of Pattern 44 which contains 4 HD4 content providers on a single EIA. The Solution Type Usage table shows that 18 total EIA slots would be open in the optimized RF channel map and 28 additional digital audio channels could be accommodated within the set of patterns chosen by the model.

Scenario: Traverse City Region, MI 750

EIA Channels	EIA Channels In Use	EIA Channels Not Used	Solution EIA Channels In Use	Solution EIA Channels Not Used	Solution EIA Channel Improvement	Solution EIA Channel Cost Improvement
116	111	5	98	18	13	$1,300,000

EIA Channels Not Changed	EIA Channels Changed	Total Moves	Moves Cost	Solution Net Value
79	37	35	35,000	$1,265,000

Solution Type Usage

Type	Used Capacity	Unused Capacity	Unmet Demand
SD	138	0	0
SD2	0	0	0
HD	0	0	0
HD2	6	0	0
HD3	31	0	0
HD4	121	0	0
DA	44	28	0
DOCSIS 3.0	24	0	0
DOCSIS 3.1	0	0	0
SDV	8	0	0
VOD	6	0	0
SDV-VOD	0	0	0
RESERVED	5	0	0
OPEN	0	18	0

Optimal Patterns

Pattern Name	Pattern Count
Pattern44:0-0-4-0-0	28
Pattern81:DOCSIS3.0	24
Pattern86:Open	18
Pattern17:0-3-0-0-0	9
Pattern83:SDV	8
Pattern84:VOD	6
Pattern85:Reserved	5
Pattern67:0-0-0-13-0	4
Pattern56:0-0-1-10-0	3
Pattern01:2-0-0-0-0	2
Pattern68:0-0-0-12-8	2
Pattern05:1-0-2-0-0	1
Pattern10:1-0-0-6-0	1
Pattern18:0-2-1-1-0	1
Pattern25:0-1-2-2-0	1
Pattern34:0-1-0-9-0	1

Figure 12 - Optimization Results Summary Table Example

While a standard summary table and revised RF channel map can fully define optimization system results, further visualization is important when exercising the optimization system iteratively and when attempting to identify the best solution from among alternative optimal solutions. Results visualization is also very important to gaining acceptance of optimized solutions, quanitifying benefits, and recognizing the impact that changes will have on moving to an optimized re-allocation.

There are two sets of visualizations that are particularly beneficial – visualization of detailed content moves/re-assignments, and visualization of before/after EIA slot allocations across the full plant spectral bandwidth.

Detailed content changes required to implement an optimized configuration can be visualized at a high-level using a Sankey diagram. A sample Sankey diagram is shown in Figure 13.

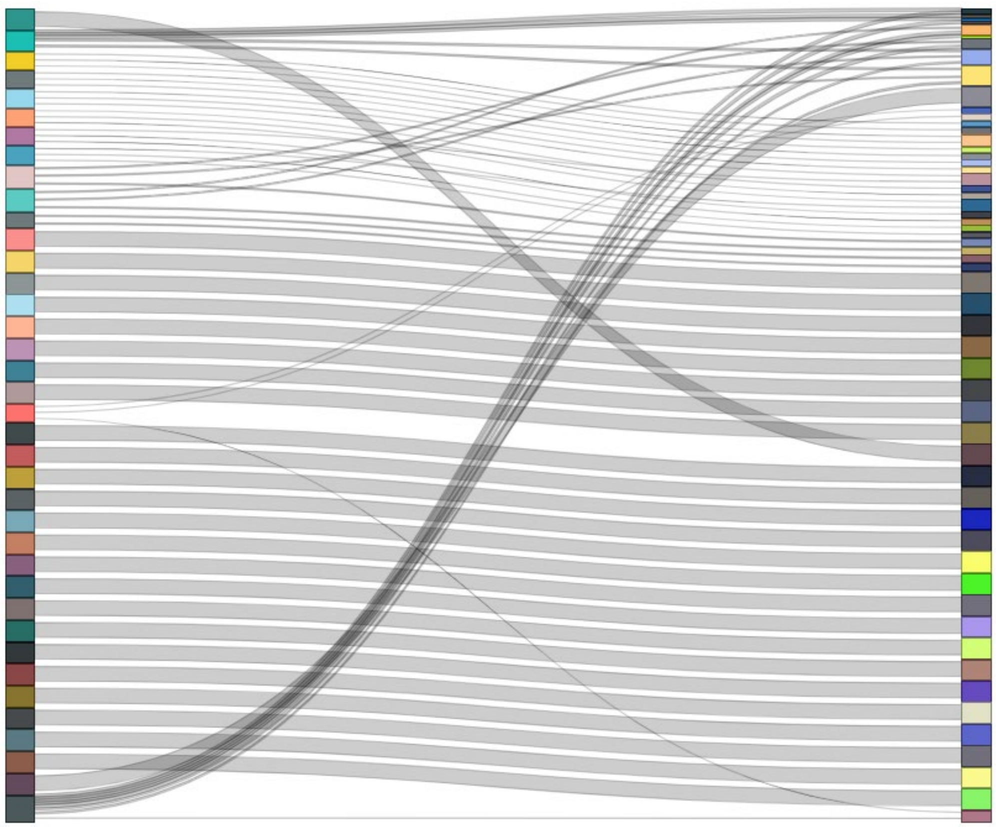

Figure 13 - Sankey Diagram of Content Moves Required

The Sankey diagram identifies visually each of the changes that would be required (individual lines on the diagram) and the relative amount of bandwidth associated with each change (width of each line).

Low-level detailed content changes can best be visualized in tabular form. A sample portion of a move/re-assignment table for an RF channel map is shown in Figure 14.

From	To	Program Name	Program Type	Bandwidth Lineup
From 0029	To 0019	IndiePlex HD East	HD4	9.5
From 0029	To 0026	movieplex HD	HD4	9.5
From 0029	To 0051	RetroPlex HD	HD4	9.5
From 0030	To 0043	ABC Family HD East	HD4	9.5
From 0030	To 0046	MTV HD East	HD4	9.5
From 0030	To 0055	Nick HD East	HD4	9.5
From 0031	To 0024	CNN HD	HD3	12.8
From 0031	To 0042	Discovery Channel HD East	HD3	12.8
From 0031	To 0095	USA Network HD East	HD3	12.8
From 0032	To 0011	Animal Planet HD	HD3	12.8
From 0032	To 0012	Big Ten Network HD	HD3	12.8
From 0032	To 0056	TLC HD East (The Learning Channel)	HD3	12.8
From 0033	To 0028	History HD East	HD4	9.5
From 0033	To 0038	FX HD East	HD4	9.5
From 0033	To 0055	National Geographic Channel HD East	HD4	9.5
From 0034	To 0040	Bravo HD East	HD4	9.5
From 0034	To 0044	MSNBC HD	HD4	9.5
From 0034	To 0098	truTV HD East	HD4	9.5
From 0035	To 0045	ESPN 2 HD	HD3	12.8
From 0035	To 0053	ESPN HD	HD3	12.8
From 0036	To 0014	AMC HD East	HD4	9.5
From 0036	To 0049	Spike TV HD East	HD4	9.5
From 0036	To 0097	Travel Channel HD East	HD4	9.5
From 0070	To 0013	SundanceTV HD East	HD4	9.5
From 0070	To 0048	Smithsonian Channel HD	HD4	9.5
From 0070	To 0098	Velocity HD	HD4	9.5
From 0071	To 0011	HBO Signature East	SD	2.8
From 0071	To 0011	MoreMax East	SD	2.8
From 0071	To 0024	Showtime 2 East	SD	2.8
From 0071	To 0024	Showtime Extreme East	SD	2.8
From 0071	To 0024	Showtime Showcase East	SD	2.8
From 0071	To 0024	SundanceTV East	SD	2.8
From 0071	To 0024	TeenNick	SD	2.8
From 0071	To 0025	HBO 2 East	SD	2.8

Figure 14 - Detailed Content Re-Assignment Table Sample

The table shows each individual content assignment's original location in the first column (From) and its optimized location in the second column (To).

Before/after visualization of full plant spectral bandwidth can be viewed using bar charts. Using bar charts, the bandwidth differences between current RF channel assignments and optimization alternative assignements can be quickly visualized.

A bar chart format shows how the full spectrum is being utilized and the location of content assignments relative to each other.

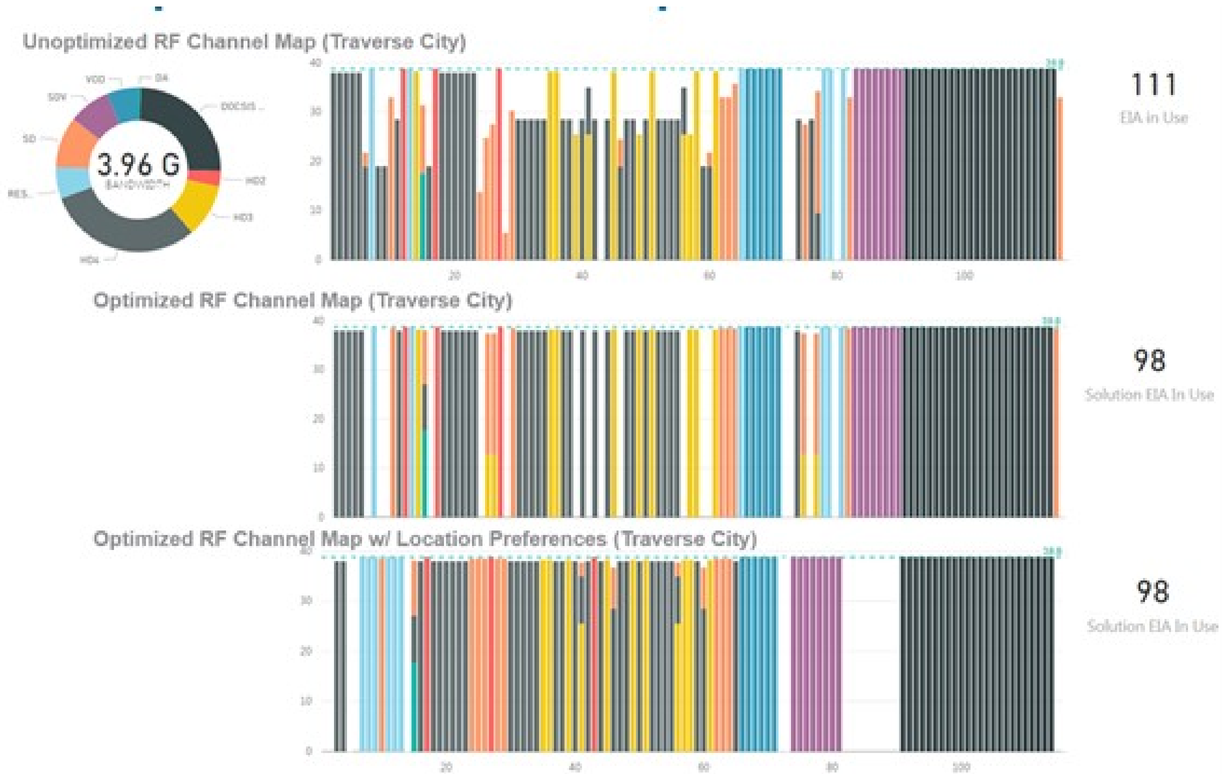

Figure 15 - EIA Slot Visualization Example

Figure 15 shows an example of how EIA bar chart visualization can be used to identify differences in channel map allocations.

The top bar chart of Figure 15 shows the initial content assignments of a channel map which was using 111 of 116 available EIA slots. The different types of content are identified by color. The height of each bar indicates how much of each 6 MHz EIA slot's capacity is being utilized.

The middle bar chart shows a minimum EIA slot assignment which has freed up 13 additional EIAs by reassigning content – using only 98 of the 116 available EIA slots.

The lower EIA bar chart of Figure 15 shows an alternative minimum EIA slot configuration that has been biased toward placing content in alignment with a desired standardized preference template – while still adhering to the minimum number of EIA slots. The preference template is defined by the Preferences Control Table of Figure 9.

Viewing the three bar charts in combination, comparisons can be made to see how much EIA utilization has improved with the EIA slot minimization, how much content has shifted positioning in the spectrum, and how much the optimized assignments were able to adhere to the engineering preferences defined in the control tables.

3.5. Execution Modes

The RF bandwidth optimization model is designed to be executed according to the data flow shown in Figure 16.

Data blending software is used to calculate the number of moves that will be required if an individual pattern is placed on an existing EIA slot. It is also used to generate the bandwidth demand by content type from an original Operational RF Channel Map. It is also used to create the mathematical matrix input to the commercial mixed integer programming optimizer.

The MILP (Mixed Integer Linear Programming) commercial optimizer then optimizes the model and produces a set of tables reflecting an optimized RF Channel Map and its associated chosen solution summary results.

Visualization tools then place optimization results into summary results tables, Sankey diagrams, and bar spectral bandwidth bar charts for analysis.

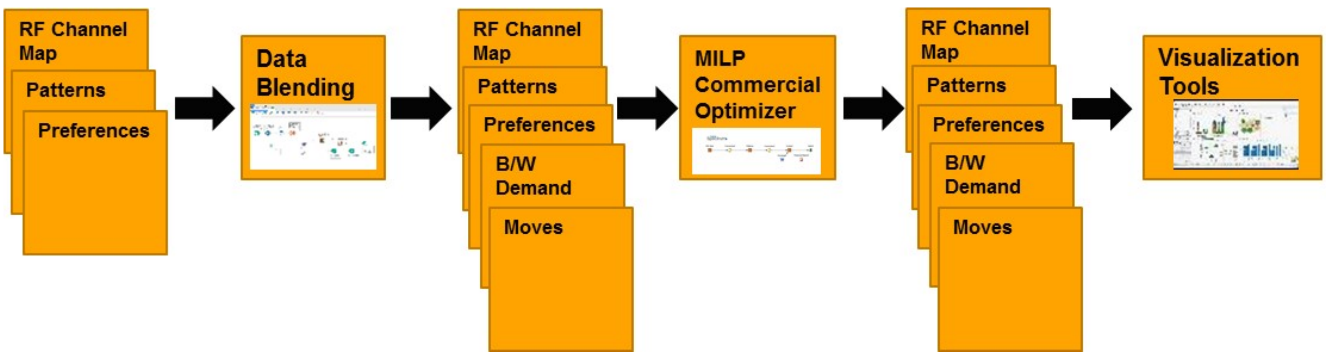

Figure 16 - Data Flow Example

4. Business Integration and Automation

In order to effectively integrate the optimization modeling system into the business there are four primary requirements – accurate automated RF channel maps, IP data data traffic demand, video traffic demand, and engineering architectural requirements/direction.

The RF Channel Map (all content assignments in each 6 MHz slot of the spectrum) requires consolidation of allocations to linear video, switched digital video, DOCSIS 3.0 SC-QAM, DOCSIS 3.1 OFDM blocks, and channels reserved for special uses like CLI. Integration of multiple data sources is required. Without automation, the time required to generate and maintain RF Channel Map currency creates a major hurdle that must be overcome to effectively optimize bandwidth.

Accurate prediction of IP data traffic demand is important in order to integrate bandwidth optimization modeling into the business. Peak traffic CMTS utilization can give a good view of current use, but with the movement to more IP video and changing data profiles, it is becoming important to use MAC-level traffic analysis from sources like IPDR to produce accurate forecasts of traffic growth. MAC cable modem mix (e.g. D3.0 vs. D3.1) and individual subscriber segmentation are becoming more and more important in assessing traffic volume requirements on individual service groups.

Accurate prediction of video traffic demand is similarly necessary integrate bandwidth optimization into the business. Compression techniques like MPEG4, increased movement toward switched digital pooling, migration of subscribers to IP video (e.g. cord cutters), and even migration of linear video products to IP video make accurate assessment of bandwidth demand increasingly difficult.

Tying in to the engineering plans for bandwidth architecture changes, new DOCSIS software releases, bandwidth reconfigurations like upstream mid-splits and/or high-splits, etc. is another key need for successful business integration and automation. Synchronization of capacity planning across operations, engineering, video, and data organizations is needed for bandwidth optimization to be effective.

Conclusions

The three major conclusions that can be drawn regarding use of MILP in optimizing spectral bandwidth are:

1. Data acquisition and data management are the biggest hurdles and most critical success factors.
2. Mixed integer linear programming is a very viable approach to optimizing bandwidth, and
3. There are many benefits to be gained through optimizing spectral bandwidth.

RF Channel Maps containing the contents/allocation of each EIA slot for each plant are necessary for any modeling or optimization. Documentation of preferred standardized configurations and engineering preferences also need to be compiled. Storage in an automated, accessible repository provides high returns in terms of reducing the time required (both modeling and manual manipulation) to analyze bandwidth use.

Mixed integer linear programming optimization for spectral bandwidth optimization requires very little compute time and model scenarios can be turned quickly. This is important when solving for all plants across the corporation as well as iteratively working to find the best optimal configuration associated with an individual channel map. Compute time is generally less than 5 seconds to evaluate as many as 2^{5000} combinations of content assignments to slots. It can also be concluded that MILP can be used without mathematical programming knowledge when designed with a data-driven approach.

Finally, and most importantly, there are many benefits to be had by optimizing spectral bandwidth. With the move to digital, packing program content can open up a lot of bandwidth – avoiding plant upgrades and accommodating unanticipated IP data traffic growth. This is especially important since the value of 6MHz of bandwidth is large and continuing to grow. Benefits in terms of time savings also accrue with optimization modeling. The time required to maintain RF Channel Maps and manipulate configurations to determine the feasibility of upgrades can be reduced. The time required to analyze strategic engineering alternatives can also be greatly reduced.

Abbreviations

5GL	Fifth Generation Language
CLI	Cumulative Leakage Index
CMTS	Cable Modem Termination System
DOCSIS	Data Over Cable Service Interface Specificationw
EIA	Electronic Industries Association
IPDR	Internet Protocol Data Record
MAC	Media Access Control
MHz	Megahertz
MILP	Mixed Integer Linear Programming
MSO	Multiple-Sytems Operator
OFDM	Orthogonal Frequency Division Multiplexing
RF	Radio Frequency
SC-QAM	Single Carrier Quadrature Amplitude Modulation

Bibliography & References

ARRIS Cable Technician Pocket Guide Subscriber Access Networks, Document MX0398 Revision P; ARRIS Enterprises, Inc., 2014, pp. 2-1,2-37.

CableLabs Data-Over-Cable Service Interface Specifications DOCSIS®3.1, MAC and Upper Layer Protocols Interface Specification, CM-SP-MULPIv3.1-I18-190422.
https://specification-search.cablelabs.com/CM-SP-MULPIv3.1

CableLabs Data-Over-Cable Service Interface Specifications DOCSIS®3.1, CCAP™ Operations Support System Interface Specification, CM-SP-CCAPv3.1-I15-190422.
https://specification-search.cablelabs.com/CM-SP-MULPIv3.1

Fredrick S. Hillier and Gerald J. Lieberman, *Introduction to Operations Research*, McGraw-Hill, tenth edition, 2015, ISBN: 0073523453. 1010 Pages.

