FM Induced Noise in Analog Fiber Optic CATV Links

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ABSTRACT

This paper derives a model for the CNR in the presence of un-modulated FM and TV carriers on an analog fiber optic CATV link. Empirically, the inter-modulation distortion generated by the FM carriers mixing with themselves and with the TV carriers cannot be distinguished from noise and therefore in the presence of FM carriers, a new noise contribution should be added to the CNR formula denominator, namely distortion induced FM noise. It is the specific objective of this paper to derive the a modified CNR equation taking into account the FM effects.

DISTORTION INDUCED FM NOISE

The conventional carrier to noise ratio (CNR) formula for an analog lightwave link is given in Eq. 1:

$$\mathrm{CNR} = \frac{\mathrm{Carrier}}{\mathrm{Noise}} = \frac{\mathrm{Carrier}}{\mathrm{Shot} + \mathrm{Thermal} + \mathrm{RIN}} \tag{1}$$

In the presence of FM carriers, a new noise contribution should be added to the CNR formula denominator, namely the distortion induced FM noise:

$$\mathrm{CNR} = \frac{\mathrm{Carrier}}{\mathrm{Shot} + \mathrm{Thermal} + \mathrm{RIN} + \mathrm{FM}} \tag{2}$$

The normalized photocurrent in the receiver (including up to third order distortion terms and normalizing by the DC current generated by the CW light) is given by:

$$i = \phi + \alpha_2\phi^2 + \alpha_3\phi^3 \tag{3}$$

where:

$$\phi = m_{am}\sum_i \cos(\omega_i t + \psi_i) + m_{fm}\sum_j \cos(\omega_j t + \psi_j) \tag{4}$$

is the RF input (AM+FM carriers). Typically, the per carrier AM modulation index (m_{am}) is 4 dB higher than its FM counterpart (m_{fm}). For clarity we will refer to the modulation index simply as *m*, the nature of which (m_{am} or m_{fm}) should be clear from the context.

The powers of the various distortion components (proportional to the squares of the currents) are given by expressions proportional to α_2 and m^4 for the CSO and α_3 and m^6 for the CTB terms, where α_2 and α_3 are the second and third order Taylor coefficient of the memoriless nonlinearity modelling the link, respectively. While it would be possible to evaluate α_2 and α_3 directly from CSO and CTB measurements at a given modulation index for a known TV frequency plan, this is really not necessary if the composite distortions due to the AM and FM+AM frequency plans are known.

The general functional dependence of the new FM "noise" term is given by:

$$\mathrm{FM} = F_2 m^4 + F_3 m^6 \tag{5}$$

where F_2, F_3 are constants to be determined below and the two terms correspond to the CSO and CTB involving the FM+TV channels. Once the general functional dependence of the various terms on *m* is understood one can write a general expression for the total CNR (renaming F2 and F3 - CNR_{cso} and CNR_{ctb}, respectively):

$$\mathrm{CNR}_{\mathrm{tot}} = 10 Log_{10} \frac{m^2}{m_o^2 \left(\mathrm{cnr}_o^{-1} + \mathrm{cnr}_{\mathrm{CSO}}^{-1} \left(\frac{m}{m_o}\right)^4 + \mathrm{cnr}_{\mathrm{CTB}}^{-1} \left(\frac{m}{m_o}\right)^6\right)} \quad (6)$$

In this expression m_o is a nominal modulation index at which the CNR in the absence of FM effects is known to be equal to the nominal CNR_o at the modulation index m_o as obtained using the regular CNR formula (Eq. 1).
The next two terms correspond to the distortion power contributions represented as equivalent noise, i.e. $cnr_{\mathrm{CSO}} = 10^{\mathrm{CNR}_{\mathrm{CSO}}/10}$ and $cnr_{\mathrm{CTB}} = 10^{\mathrm{CNR}_{\mathrm{CTB}}/10}$ are the FM-CSO and FM-CTB induced CNR contributions (in linear scale), respectively. Since m_o^2 is the (normalized) carrier power, then these expressions represent the (normalized) noise power contributions due to FM-CSO and FM-CTB respectively in the video channel bandwidth.

To determine these terms theoretically, one makes use of a beat counting or distortion modelling program and runs it at an arbitrary modulation index for both TV channels alone and FM+TV channels. The two distortions distributions are subtracted from each other to find the contribution of the FM terms alone (this is done for CSO and CTB separately).

Let $\mathrm{ctb}_{\mathrm{TV}}(\omega_i)$, $\mathrm{ctb}_{\mathrm{FM+TV}}(\omega_i)$ be the two (linear scale) CTB power profiles for frequency plans involving TV channels and FM+TV channels, respectively. For a given channel frequency ω_i, Eq. 7 represents the "FM alone" CTB contributions, actually comprising all beats which involve at least one FM channel, e.g. in the CTB case, triple beat products involving one or two or three FM channels and two or one or no TV channels respectively.

$$\mathrm{ctb}_{\mathrm{FM}}(\omega_i) = \mathrm{ctb}_{\mathrm{FM+TV}}(\omega_i) - \mathrm{ctb}_{\mathrm{TV}}(\omega_i) \quad (7)$$

The contribution of all triple products involving TV channels alone is deleted by the subtraction. A similar procedure is performed to find the "FM alone" CSO contributions. The net "FM alone" composite beat profiles are now integrated over the bandwidth of the observation channel, using a noise equivalent bandwidth B_{TV} of 5 MHz for PAL or 4 MHz for NTSC.

These procedures yield the total distortion power in the channel band. In order to calibrate the scale, the beat profile program distortion results for the TV channels alone are used in the following way (for definiteness the CTB case is discussed, a similar reasoning applies to the CSO as well):
Let $\mathrm{ctb}_{\mathrm{TV}}(\omega_o)$ be the (linear scale) CTB power as generated by the beats counting program at the reference channel frequency ω_o where the CTB performance is specified to be equal to $\mathrm{CTB}_{\mathrm{TV}}(m_o)$ [dB] at the nominal modulation index m_o. Then

$10\log_{10}\left(\frac{\mathrm{ctb}_{\mathrm{FM}}^{(\mathrm{ch})}}{\mathrm{ctb}_{\mathrm{TV}}(\omega_o)}\right)$ is a figure describing the level of the integrated FM-CTB relative to the TV-CTB. The integrated FM-CTB is therefore (in dBc):

$$-\mathrm{CNR}_{\mathrm{CTB}} = \mathrm{CTB}_{\mathrm{TV}}(m_o) + 10\log_{10}\left(\frac{\mathrm{ctb}_{\mathrm{FM}}^{(\mathrm{ch})}}{\mathrm{ctb}_{\mathrm{TV}}(\omega_o)}\right) \quad (8)$$

A similar expression applies to the CNR contribution due to CSO.

In the next section, we will use an example to demonstrate how to apply the analysis to a specific AM+FM channel plan.

EXAMPLE

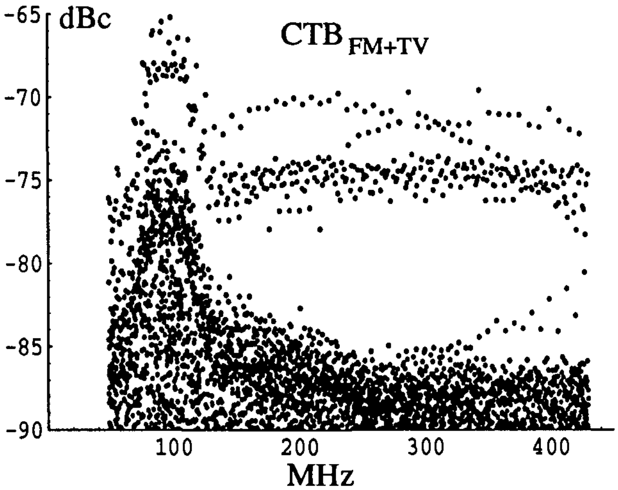

Fig. 1: Total induced CTB vs. frequency (35 PAL channels and 30 FM channels)

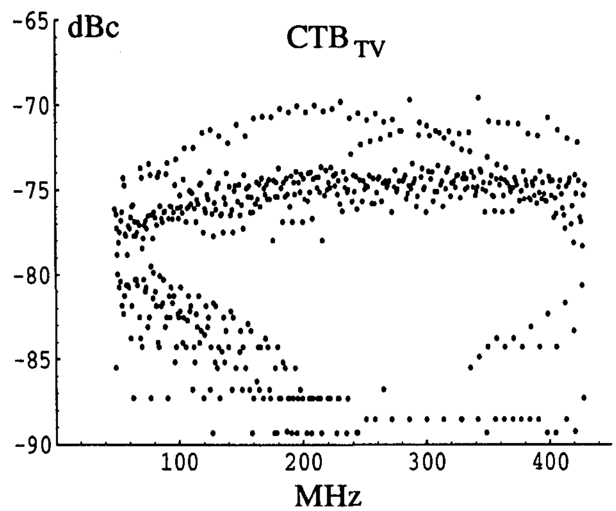

Fig. 2: CTB vs. frequency (35 PAL channels)

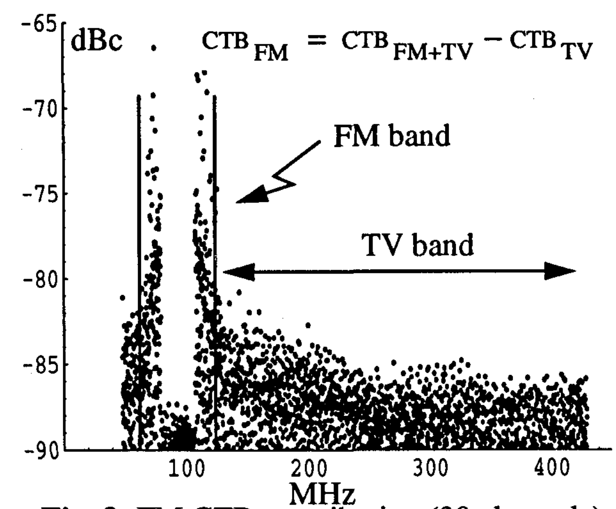

Fig. 3: FM CTB contribution (30 channels)

We illustrate the foregoing analysis by using an example. The TV plan chosen includes 35 PAL channels at 3.5% index modulation (m_{am}) each and 30 FM carriers (inserted between 80 and 108 MHz) with modulation index (m_{fm}) of 2.2% (4 dB below the AM). A beat counting program was used to generate the CSO and CTB profiles of the combined TV+FM channels and of the TV channels only. The FM channels contribution to either case (CSO and CTB) was obtained by calculating the difference between the two outcomes.

Fig. 1 through Fig. 3 depict the calculation results for the CTB case. First, the total induced CTB profile of both the FM and TV channels is calculated (Fig. 1), Fig. 2 depicts the CTB profile of the TV channels only and the difference between the two which yields the CTB contribution of the FM channels is shown in Fig. 3. The contribution of the FM channels to the CTB profile at the TV channels frequency band is clearly shown in the Figure.

The noise-like nature of the FM contribution can be seen in a close-up view of Fig. 3. To achieve that, an observation channel is arbitrarily chosen at mid-band at a frequency of 294.25 MHz. The CTB contribution of the FM channels over the 5 MHz filter bandwidth is shown in Fig. 4. Notice that the CTB contribution is scattered

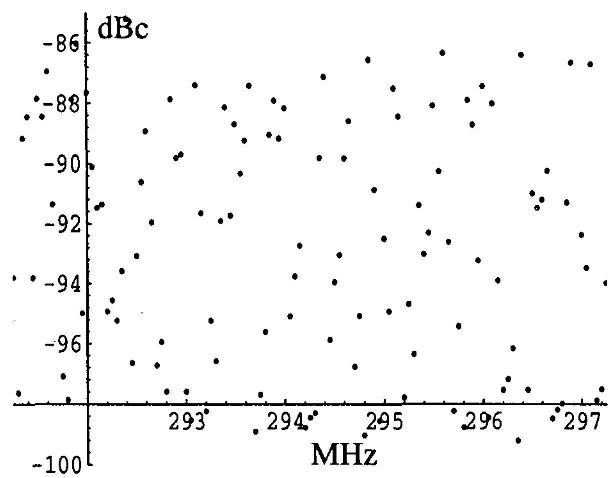

Fig. 4: CTB due to FM over the PAL observation channel (carrier at 294.25 MHz).

in random manner over the entire channel band at a level which is about -85 dBc. A similar noise-like behavior is observed for the CSO contribution of the FM channels.

The CNR equivalent degradation of the FM channels due to their CTB (CNR_{ctb}) is obtained by using Eq. 8. Integrating the CTB contribution over the channel filter (containing in this case 100 discrete points) and using a CTB_{TV}(294.25 MHz)=-71 dBc (see Fig. 2) we obtain (following similar procedure for CSO):

$$\begin{aligned} CNR_{ctb} &= 62.1\,\mathrm{dB} \\ CNR_{cso} &= 70.4\,\mathrm{dB} \end{aligned} \tag{9}$$

At the distortion levels shown above (CTB=-71, CSO=-70), the CNR_0 at PAL filter bandwidth of 5 MHz is expected to be 51.4 dBc.

To understand the significance of the above numbers let us plot the CNR expression of Eq. 6 using the numbers obtained in Eq. 9.

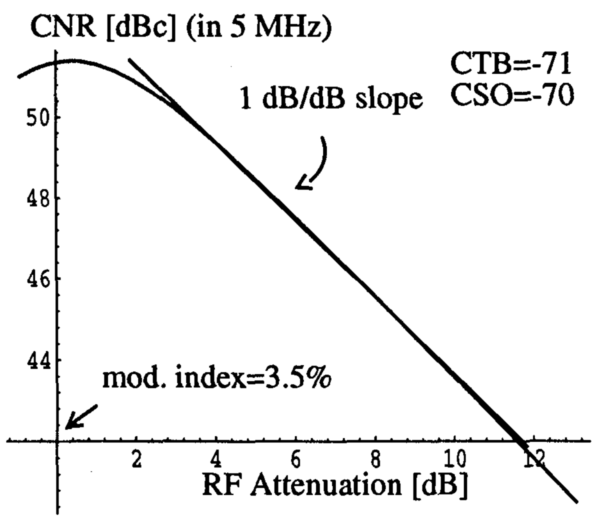

Fig. 5: CNR vs. RF attenuation including FM induced noise. Notice the deviation from straight line as modulation index increases.

As is shown in the Figure, the predicted degradation of the FM induced noise will cause a compression in the CNR performance with increase in modulation index. In contrast, the theoretical expression for CNR in Eq. 1 predicts that the CNR will follow the RF attenuation on a dB/dB basis. From Fig. 5 for example, the FM induced noise is expected to degrade the theoretical CNR performance ("FM less") by as much as 0.5 dB at a modulation index of 3.5%

COMPARISON WITH EXPERIMENT

To confirm the theoretical prediction of the FM noise contribution, an externally modulated YAG transmitter (Harmonic Lightwaves model HLT 6720) was loaded with the same 35 PAL channels and its CNR was measured as a function of the RF input pad with and without FM channels loading. The results and a comparison with the theoretical prediction shown in Fig. 5 are summarized in Fig. 6.

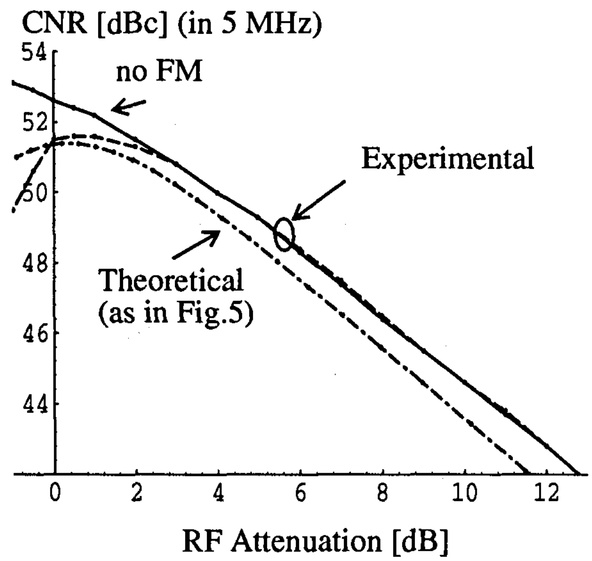

Fig. 6: A comparison between experimental and theoretical CNR vs. RF attenuation. Two experimental curves are shown, one including FM channels and the other "FM less".

The experimental results for the "FM less" case follow a straight line but the CNR curve starts bending in the region corresponding to modulation index of 4.5%. This seems to indicate clipping induced noise, a mechanism not taken into account in the present analysis. The predicted effect of FM loading on the CNR performance is clearly seen in the Figure by the deviation of the FM loading experimental curve from its "FM less" counterpart. There is an excellent agreement between the theoretical and experimental results over most of the RF range. The excessive compression of the FM loading experimental results evident at high modulation index (at attenuation levels less than 0 dB), can be attributed to higher then third order effects. Recall that the assumptions of the model presented here include only the effects of second and third order distortion, no higher orders or clipping distortion are taken into account.

CONCLUSION

Using non-linear (second and third order) analysis we have developed a model describing the effects of FM channel loading on a fiber-optic analog link. The FM channels loading is shown to degrade the CNR performance due to the non-linear mixing of the FM channels with themselves and with the AM channels resulting in an increase in the noise floor of the optical link. A corrected CNR equation taking into account the distortion induced FM noise is proposed.

To confirm the theoretical prediction a test which was comprised of 35 PAL channels and 30 FM channels was carried out and produced excellent agreement. As predicted, the CNR deviated from its 1 dB/dB slope due to FM channels loading as the optical modulation index increased. At very high modulation indexes the CNR was compressed even further than theoretically predicted indicating higher order non-linear contributions, contributions not included in the presented model.