# DISTORTION ACCUMULATION IN TRUNKS

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## ABSTRACT

*The conventional analysis of distortion in a cable television system assumes that composite triple beat (CTB) accumulates on a voltage basis, which is a worst case situation. This assumption, although it may be valid for long cascades of identical devices, such as push-pull trunk amplifiers, may not be valid for feedforward trunk cascades. This is because feedforward amplifiers, which use distortion cancellation, may not be identical to other feedforward amplifiers in the cascade.*

*Test results for pairs of push-pull and feedforward CATV equipment are presented, as well as a theoretical analysis which explains these results.*

## INTRODUCTION

Many papers have been published discussing the cascading of CTB in cable television trunks (1-4). These papers indicate that CTB cascades with voltage (20logN) addition, the worst case possibility. CTB is generally considered to be the limiting intermodulation distortion in a cable television trunk. Consequently, the 20logN assumption has a large impact on cable system architecture and cost.

In order for 20logN addition to occur, two basic conditions must be met in an amplifier: 1) a linear phase vs. frequency response and 2) identical distortion phases for all amplifiers. Both of these criteria are met in a push-pull cascade, and, therefore, 20logN cascading results. Although feedforward amplifiers have a linear phase vs. frequency response, it is not clear that they all have equal distortion phases. Unlike the distortion phase of a push-pull amplifier, the distortion phase of a feedforward amplifier is heavily dependent on the amplitude and phase matching of signals when the distortion from the main amplifier is cancelled by signals from the error amplifier, so it is expected that distortion phase is not constant from device to device. If distortion phases vary significantly, the 20logN assumption is very pessimistic.

## CASCADING OF CTB

Consider the repeater section in Exhibit 1

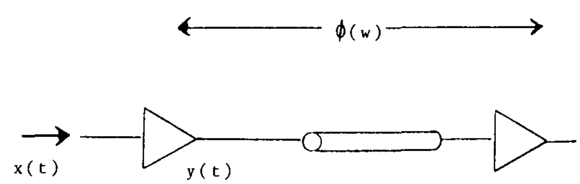



EXHIBIT 1 - Repeater Section

The input signal is x(t) and the output is y(t) which is represented by:

$$y(t) = K_1 x(t) + K_2 x^2(t) + K_3 x^3(t) + ....$$

Note that frequency dependent non-linear characteristics are ignored. A more rigorous treatment is possible using a Volterra series, but this makes the analysis much more complex. Higher than third-order distortions are also ignored.

Let the input function x(t) consist of three amplitude-modulated sinusoidal voltages:

$$x(t) = A(t)\cos w_a t + B(t)\cos w_b t + C(t)\cos w_c t$$

The output y(t) consists of many terms, including a triple beat term:

$$3/2\, K_3\, A(t)B(t)C(t) \cos(w_a t + w_b t - w_c t + \Theta_1)$$

where $\Theta_1$ is the phase of the triple beat in the first repeater at frequency a + b - c.

In a cable television system, many of these triple beats are formed, and their frequencies are nominally those of a video carrier frequency. The accumulation of beats is CTB.

Assume that the repeater section has a linear phase vs. frequency characteristic as is shown in Exhibit 2

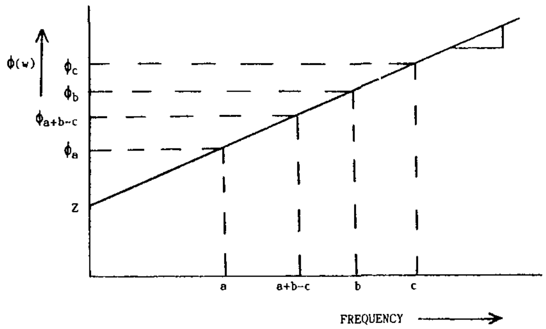



EXHIBIT 2 - Linear phase-frequency characteristic of a repeater section

The phase shift $\phi$ from the repeater section is:

$$\phi(w) = mw + Z$$

The signal at the output of the first repeater will include the terms:

$$K_1 A(t) \cos(w_a t + \phi_a)$$
$$+ K_1 B(t) \cos(w_b t + \phi_b)$$
$$+ K_1 C(t) \cos(w_c t + \phi_c)$$
$$+ 3/2 K_3 A(t)B(t)C(t) \cos (w_a t + w_b t - w_c t + \Theta_1)$$

The signals at the output of the second repeater will include the terms from the first repeater:

$$K_1 A(t) \cos(w_a t + \phi_a + \phi_a)$$
$$+ K_1 B(t) \cos(w_b t + \phi_b + \phi_b)$$
$$+ K_1 C(t) \cos(w_c t + \phi_c + \phi_c)$$
$$+ 3/2 K_3 A(t)B(t)C(t) \cos (w_a t + w_b t - w_c t + \Theta_1 + \phi_{a+b-c})$$

It will also include a term for the triple beat generated in the second repeater:

$$+ 3/2 K_3 A(t)B(t)C(t) \cos (w_a t + w_b t - w_c t + \Theta_2 + \phi_a + \phi_b - \phi_c)$$

Comparing the phases of the two triple beats shows a difference $\Theta_D$ of:

$$\Theta_D = \Theta_1 + \phi_{a+b-c} - (\Theta_2 + \phi_a + \phi_b - \phi_c)$$
$$= \Theta_1 - \Theta_2 + \phi_{a+b-c} - \phi_a - \phi_b + \phi_c$$

But, from Exhibit 2,

$$\phi_{a+b-c} = m(a+b-c) + Z$$
$$= ma + mb - mc + Z,$$
$$\phi_a = ma + Z$$
$$\phi_b = mb + Z$$
$$\phi_c = mc + Z$$

Therefore,

$$\Theta_D = \Theta_1 - \Theta_2 + ma + mb - mc + Z - ma - Z - mb - Z + mc + Z$$
$$= \Theta_1 - \Theta_2$$

If $\Theta_1$ and $\Theta_2$ are equal, then the beats will add on a voltage basis. If $\Theta_1$ and $\Theta_2$ are not equal, then the beats will add with a phase difference. In a cascade of many amplifiers, this will lead to MlogN addition where M depends on the statistical distribution of phases and may vary over a wide range.

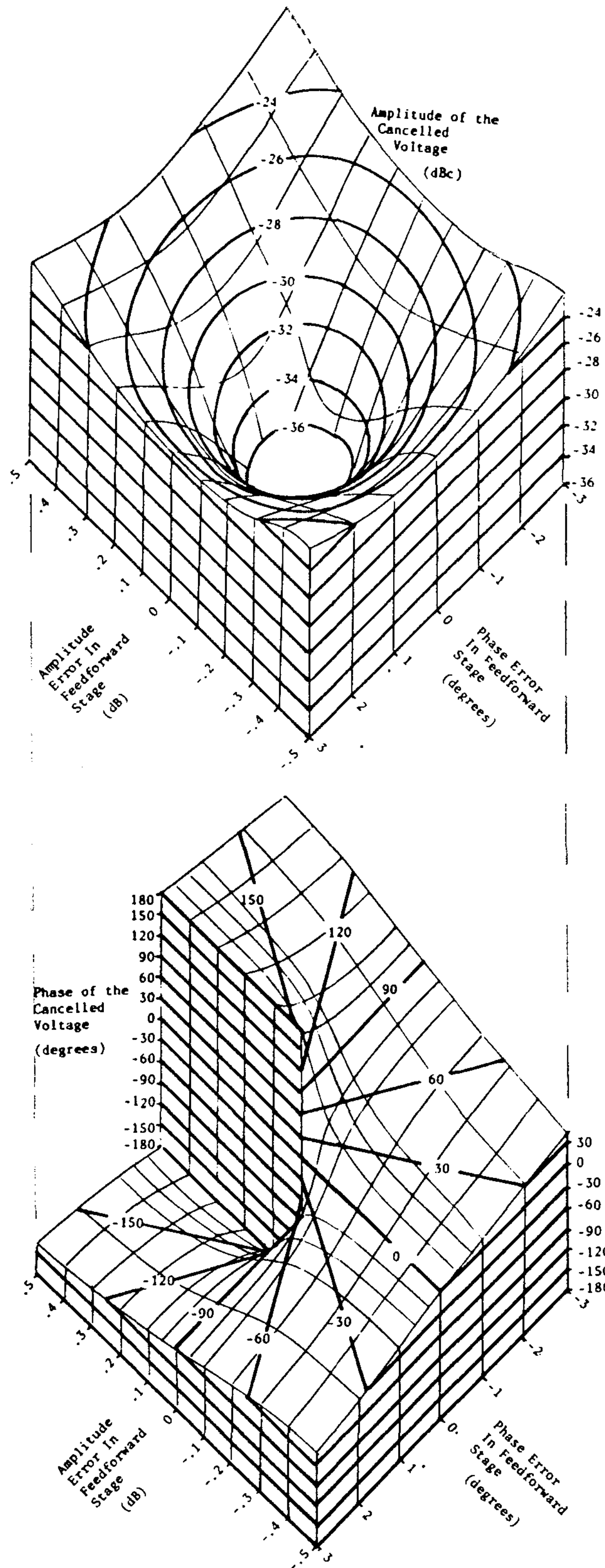
The relationship between $\Theta_D$ and MlogN for two devices in cascade is shown in Exhibit 3.

| $\Theta_D$ (Degrees) | Mlog2 Addition | Type of Addition |
|---|---|---|
| 0 | 20.0log2 | voltage addition |
| 30 | 19.0log2 | |
| 60 | 15.8log2 | |
| 90 | 10.0log2 | power addition |
| 120 | 0.0log2 | no addition |
| 150 | -19.0log2 | partial cancellation |
| 180 | -∞ log2 | complete cancellation |

EXHIBIT 3 - Phase Difference vs. Mlog2

## FEEDFORWARD DISTORTION PHASE

Unlike push-pull amplifiers, which are generally similar to each other and which consequently generate beats with nearly equal phases, feedforward amplifiers cancel the distortion, typically with a minimum of 20dB cancellation. As is shown in Exhibit 4, small amplitude or phase changes in the feedforward amplifier cancellation circuitry can lead to large changes in the amplitude and/or phase of the resultant distortion.




Distortion Amplitude and Phase vs. Cancellation Amplitude and Phase Errors

Exhibits 5A to 5C show average CTB levels plus or minus one standard deviation as a function of temperature at 55.25, 295.25 and 547.25 MHz respectively (5). The CTB level of feedforward hybrids varies significantly from unit to unit, and varies on individual units with temperature. Temperature sensitivity is lower at 55.25 and 295.25 MHz than at 547.25 MHz. The unit to unit variation is lower at 55.25 MHz than at 295.25 and 547.25 MHz.

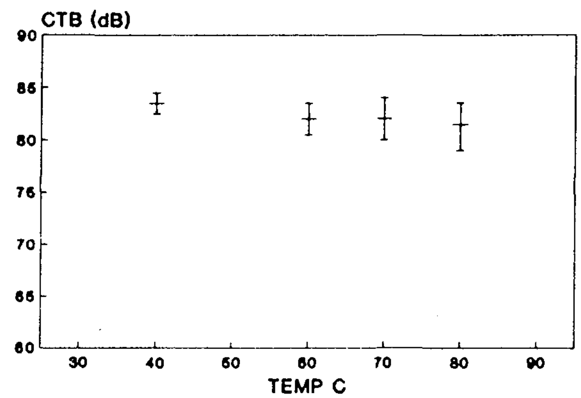



EXHIBIT 5A
CTB vs. Temperature 55.25 MHz Feedforward

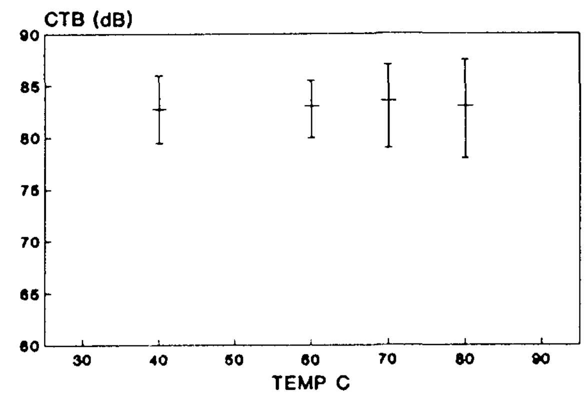



EXHIBIT 5B
CTB vs. Temperature 295.25 MHz Feedforward

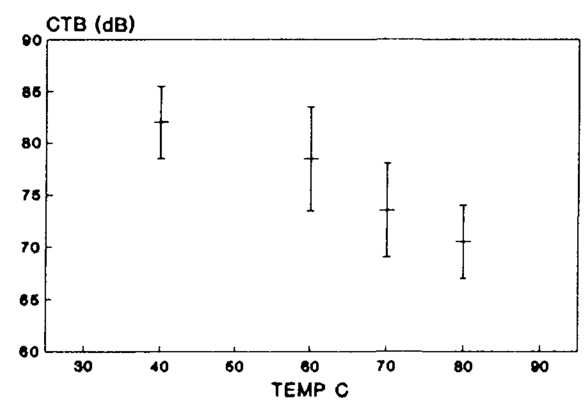



EXHIBIT 5C
CTB vs. Temperature 547.25 MHz Feedforward

Exhibits 6A and 6B show second loop cancellation as a function of termperature and frequency for two sample feedforward hybrids (6). The cancellation of feedforward hybrids varies more at 450 and 550 MHz than at 50 MHz, accounting for the greater change in CTB with temperature at higher frequencies. Note that a change in cancellation will cause an equal change in distortion level.

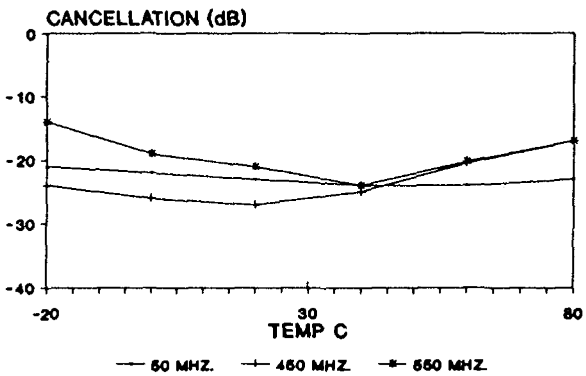



EXHIBIT 6A
2nd Loop Cancellation vs. Temp
Feedforward Sample A

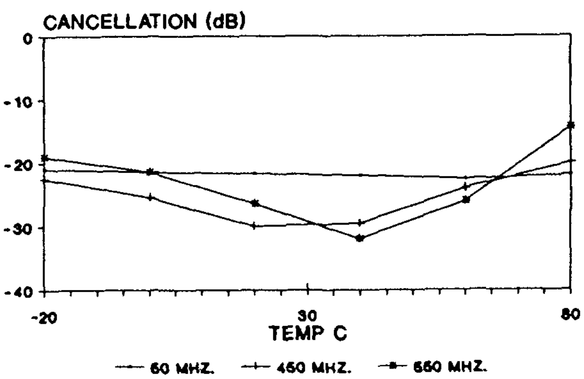



EXHIBIT 6B
2nd Loop Cancellation vs. Temp
Feedforward Sample B

The absolute distortion level is also lower at 55 MHz than at 550 MHz because the push-pull main amplifier in the feedforward circuit generates lower distortion at 55 MHz than at 550 MHz.

Most relevant to this paper is the fact that, at high frequencies, cancellation varies from unit to unit and with temperature. This leads to distortion phase variations from unit to unit and with temperature and means that feedforward amplifiers will not cascade on a 20logN basis. This concept is demonstrated with experimental data in the next section.

## TESTING OF DISTORTION PHASE

Distortion phase was measured indirectly by measuring the CTB level of one amplifier, of a second amplifier, and of the two amplifiers in cascade. Since small errors in CTB measurements can cause large errors in calculated phase values, great care was taken in making accurate measurements. Amplifiers were operated at output levels where higher than third order distortions were negligible. The output levels of amplifiers were monitored and maintained within approximately 0.1dB, whether measured singly or in cascade. The noise floor was measured and CTB levels were corrected for these noise levels.

Eight feedforward amplifiers were tested. These were Scientific-Atlanta feedforward trunk amplifiers with Motorola feedforward hybrids. The push-pull input hybrid was replaced in each module with a passive through block. As a control test, six standard Scientific-Atlanta push-pull trunk amplifiers were also tested. These amplifiers had push-pull input and output hybrids.

For each set of test results, the difference in distortion phase $\Theta_D$ was calculated. From Exhibit 7 it can be seen that

$$CTB_{TOTAL} = \sqrt{(CTB_1 + CTB_2 \cos \Theta_D)^2 + (CTB_2 \sin \Theta_D)^2}$$

where $CTB_{TOTAL}$, $CTB_1$ and $CTB_2$ are expressed as voltages, not in decibels. Manipulating this equation provides

$$\Theta_D = \cos^{-1}\left(\frac{CTB_{TOTAL}^2 - CTB_1^2 - CTB_2^2}{2\, CTB_1\, CTB_2}\right)$$

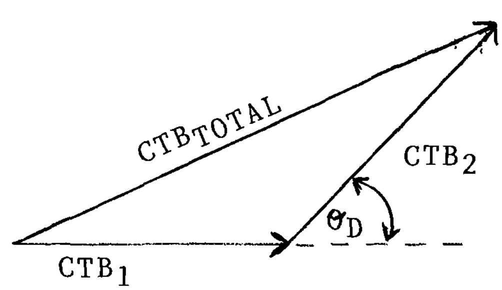



EXHIBIT 7
Vector addition of CTB

Exhibits 8A and 8B show the relative distortion phase $O_D$ of five pairs of push-pull amplifiers, at 295.25 and 445.25 MHz respectively.

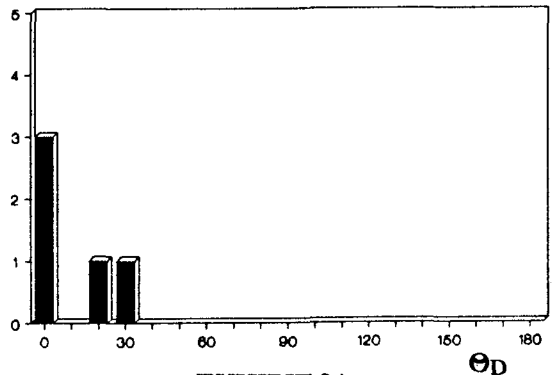



**EXHIBIT 8A**
**295.25 MHz Push-Pull Pairs**

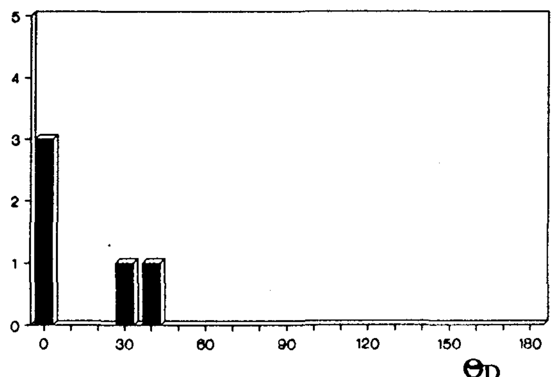



**EXHIBIT 8B**
**445.25 MHz Push-Pull Pairs**

At both frequencies, all pairs of amplifiers measured very low values of $\Theta_D$, as expected. Note that a 0.2dB error in the measured CTB will cause a calculated phase error as large as $24^o$ near $\Theta_D = 0^o$, but less than $1^o$ at $\Theta_D = 150^o$. Consequently, experimental error in the reported values of $\Theta_D$ will be greater when $\Theta_D$ is low.

Exhibits 9A and 9B show $\Theta_D$ at 67.25 and 445.25 MHz respectively, as measured on push-pull pre-amplifier and feedforward post-amplifier pairs. The distortion phases of the push-pull pre-amplifiers are all nearly identical as demonstrated above. From Exhibit 9A, the $\Theta_D$ of the feedforward amplifiers is relatively constant at this low frequency. This is because the feedforward amplifiers exhibited no deep cancellation nulls or CTB variation with temperature at low frequencies. It must be noted that although there was little variance in $\Theta_D$ at 67.25 MHz, the average value was about $120^o$, which caused no CTB degradation when push-pull and feedforward hybrids were cascaded.

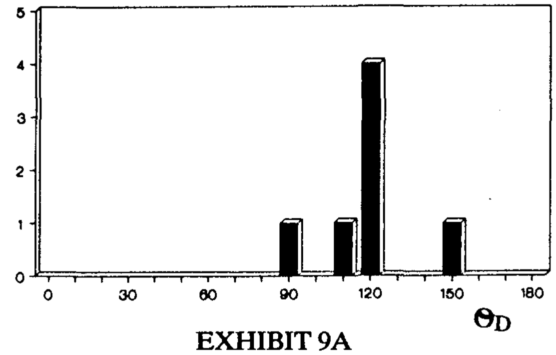



**EXHIBIT 9A**
**67.25 MHz Push-Pull Feedforward Pairs**

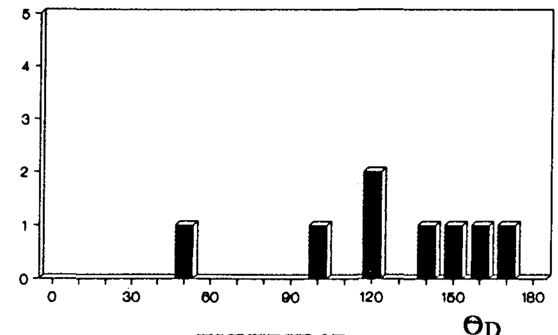



**EXHIBIT 9B**
**445.25 MHz Push-Pull Feedforward Pairs**

Results for push-pull/feedforward hybrid pairs at 445.25 MHz are shown in Exhibit 9B. Unlike at 67.25 MHz, the feedforward distortion phases vary over a wide range (from $51^o$ to $169^o$), and the units are neither generally similar to each other nor to push-pull hybrids.

From these results, one would expect that the $\Theta_D$ between feedforward hybrid pairs would have the following characteristics: at low frequencies $\Theta_D$ would be consistently low, but at higher frequencies $\Theta_D$ would tend to vary over a wide range. This was tested in the next set of results.

Distortion phase differences for pairs of feedforward amplifiers are shown in Exhibits 10A, B and C, at 67.25, 295.25 and 445.25 MHz respectively. At 67.25 MHz, $\Theta_D$ is always less than $90^o$; while at both 295.25 and 445.25 MHz, $\Theta_D$ varies over wide ranges. The results shown in Exhibits 10A to 10C demonstrate that, at intermediate and high frequencies, feedforward amplifiers will cascade on significantly less than a 20logN basis. At low frequencies, feedforward amplifiers do cascade on a near 20logN basis; however, distortion levels at this frequency are significantly lower than at higher frequencies.

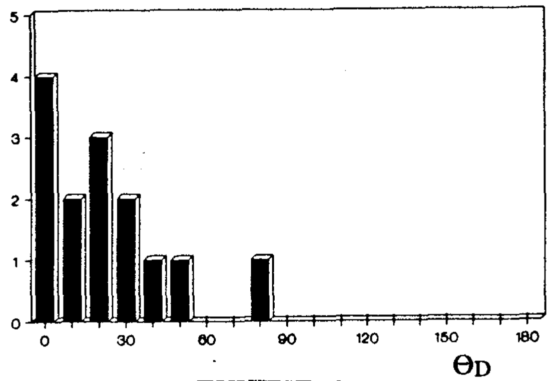



EXHIBIT 10A
67.25 MHz Feedforward Pairs

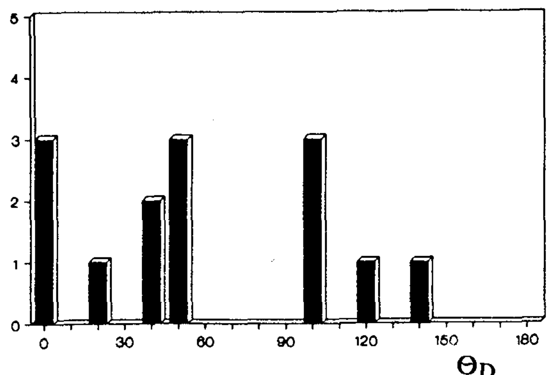



EXHIBIT 10B
295.25 MHz Feedforward Pairs

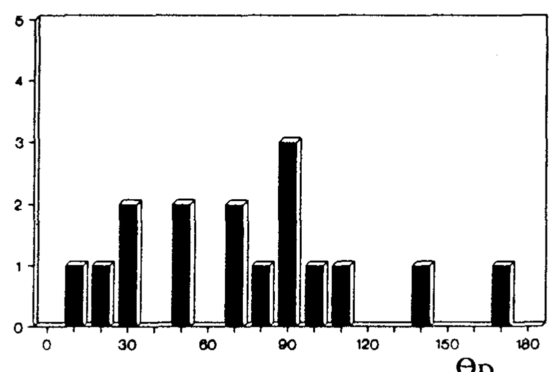



EXHIBIT 10C
445.25 MHz Feedforward Pairs

It must be noted that the number of data points for the $\Theta_D$ of feedforward amplifiers is artificially high since all the data points are not independent. The data may include the $\Theta_D$ of amplifier pairs 1 and 2, 2 and 3, and 1 and 3. If the $\Theta_D$ for two of these pairs are known, then the $\Theta_D$ for the third pair can be predicted. For example, at 445.25 MHz amplifier pairs, pairs 6 and 7 had a $\Theta_D$ of $98^o$ and pairs 7 and 8 had a $\Theta_D$ of $71^o$. Pairs 6 and 8 should, therefore, be approximately either $98-71=27^o$ or $98+71=169^o$. In fact, pairs 6 and 8 measured $174^o$, within $5^o$ of one of the predicted values.

A comparison of the predicted distortion compared to measured results shows that the angles were measured accurately (typically with less than $15^o$ cumulative error). Since the results were taken over many hours of elapsed time, they also indicate that at a constant temperature, $\Theta_D$ remains stable in a feedforward amplifier.

It is interesting to note that these phase additions apply to CTB, which is comprised of many hundreds of beats. The concept of phase addition can be applied to CTB in feedforward amplifiers because $\Theta_D$ is essentially identical for all the beats comprising the CTB. The beats generated in the main amplifiers are all similar in nature. Feedforward cancellation is performed at the final beat frequency, and all individual beats at the same approximate frequency will be cancelled identically, and will have identical phase shifts from the beats generated by the main amplifier.

Other work on cascading of CTB in dissimilar devices (e.g. AML microwave, AM fibre, and trunk amplifiers), has shown that the phase additions of CTB do not apply (7). This is the case because $\Theta_D$ varies over a wide range ($70^o$) for the individual beats comprising a CTB. Truly dissimilar devices have fundamentally different non-linear transfer characteristics and generate beats with a wide range of phases.

In feedforward amplifiers, since all beats are treated identically by the feedforward circuitry, it is possible to measure any value of $\Theta_D$, from 0 to $180^o$. Between dissimilar devices, extremely low or extremely high phase differences never occur because different beats at the same frequency add with different phases (7).

## IMPLICATIONS OF TEST RESULTS

Since CTB in feedforward amplifiers has been demonstrated to cascade with a large variation in distortion phases at high frequencies, a 20logN design rule for cable systems is excessively conservative. Worst case and expected CTB performance in a cascade of stations with feedforward post-amplifiers and push-pull pre-amplifiers is discussed below.

The push-pull pre-amplifier and feedforward post-amplifier generate approximately the same distortion level which will be taken as a reference in a worst case analysis. The single station will generate a CTB ratio 6dB worse than the reference, and a cascade of twenty stations will generate a CTB ratio 32dB worse.

Based on the previous test results, it is expected that the worst case scenario will not occur. At high frequencies, push-pull pre-amplifiers will tend to cascade randomly with feedforward post-amplifiers, generating a typical single station CTB ratio only 3dB worse than the reference. The feedforward post-amplifiers will tend to cascade randomly with each other so the cascading of the pre-amplifiers and post-amplifiers must be considered separately. A cascade of twenty pre-amplifiers will generate a CTB ratio 26dB worse while a cascade of twenty post-amplifiers will generate a CTB ratio 13dB worse. The total cascade will generate a CTB ratio 26.2dB worse, a 5.8dB improvement from the worst case calculation. Note that almost all of the system distortion is due to the push-pull pre-amplifiers.

The expected CTB ratio can also be calculated for low frequencies, where push-pull pre-amplifiers and feedforward post-amplifiers cascade with $\Theta_D = 120^{\circ}$, but where both pre-amplifiers and post-amplifiers cascade on a 20logN basis. The expected performance of individual amplifiers at low frequencies is 10dB better than at high frequencies.

These worst case and expected CTB ratios are summarized in Exhibit 11. The expected results are not recommended as a system design rule since cascade performance will demonstrate statistical scatter. More data is required to derive a system design rule.

A similar analysis of a cascade of trunk stations, with feedforward, not push-pull pre-amplifiers, is shown in Exhibit 12. With this station configuration, CTB cascades randomly at high frequencies. At low frequencies, however, 20logN cascading occurs on both the pre-amplifiers and post-amplifiers, as well as between pre-amplifier and post-amplifier pairs. The 10dB low frequency improvement still applies. Expected improvements from worst case predictions are even larger for this station configuration than for the push-pull pre-amplifier configuration: 10 and 13.8dB.

A station with a feedforward pre-amplifier would have a 3dB higher noise figure than a station with a push-pull pre-amplifier because of input losses on the error amplifier of the pre-amplifier stage. After station CTB performance has been altered by 6dB to provide a station with equivalent carrier to noise ratio (CNR), the two amplifier configurations can be compared as in Exhibit 13.

CTB RATIO RELATIVE TO REFERENCE

| | Worst Case | Expected High Freq. | Expected Low Freq. |
|---|---|---|---|
| Pre-amp | Ref. | 0 | +10 |
| Post-amp | Ref. | 0 | +10 |
| 1 Station | -6 | -3 ** | +10 *** |
| 20 Pre-amps | -26 | -26 * | -16 * |
| 20 Post-amps | -26 | -13 ** | -16 * |
| 20 Stations | -32 | -26.2 ** | -16 *** |
| Improvement from Worst Cast (dB) | -- | 5.8 | 16 |

* Assumes $\Theta_D = 0^{\circ}$
** Assumes $\Theta_D = 90^{\circ}$
*** Assumes $\Theta_D = 120^{\circ}$

EXHIBIT 11
CTB Ratios of a Cascade of Push-Pull Pre-Amplifiers, Feedforward Post-Amplifiers

CTB RATIO RELATIVE TO REFERENCE

| | Worst Case | Expected High Freq. | Expected Low Freq. |
|---|---|---|---|
| Pre-amp | +20 | +20 | +30 |
| Post-amp | Ref. | 0 | +10 |
| 1 Station | -0.8 | 0 ** | +9.2 * |
| 20 Pre-amps | -6 | +7 ** | +4 |
| 20 Post-amps | -26 | -13 ** | -16 * |
| 20 Stations | -26.8 | -13 ** | -16.8* |
| Improvement from Worst Cast (dB) | -- | 13.8 | 10 |

* Assumes $\Theta_D = 0^{\circ}$
** Assumes $\Theta_D = 90^{\circ}$

EXHIBIT 12
CTB Ratios of a Cascade of Feedforward Pre-Amplifiers, Feedforward Post-Amplifiers

| | Push-Pull Pre-amp<br>Feedforward Post-amp | Feedforward Pre-amp<br>Feedforward Post-amp |
|---|---|---|
| Low Freq. | Ref. -16 | Ref. -22.8 * |
| High Freq. | Ref. -26.2 | Ref. -19 * |

* Corrected for equal CNR

EXHIBIT 13
Expected CTB Ratios from Cascades of 20 Amplifiers

After CNR corrections, a dual feedforward cascade is expected to perform over 3dB better in CTB than a conventional cascade. Performance of such a cascade would be limited by the CTB ratio at low frequencies, because cascading tends to be 20logN at low frequencies.

## CONCLUSIONS

The phase differences between CTB distortion generated by feedforward amplifiers have a wide variation at high frequencies, causing cascading which appears to be random. At low frequencies, the phase differences are small, causing near voltage addition of CTB between feedforward amplifiers. Feedforward amplifiers and push-pull amplifier pairs typically generate CTB with $120^{\circ}$ phase difference at low frequencies.

The performance of a cascade of feedforward amplifiers will be significantly better than that predicted using 20logN addition, even in stations with push-pull pre-amplifiers. At high frequencies better performance is due to random cascading, while at low frequencies it is due to individual station CTB performance significantly better than specification.

Because of the random cascading of feedforward amplifiers at high frequencies, feedforward technology is more attractive than was previously thought. For example, a cascade of trunk amplifiers with feedforward pre-amplifiers, as well as feedforward post-amplifiers, will generate significantly less CTB than a cascade of amplifiers with push-pull pre-amplifiers and feedforward post-amplifiers. A dual feedforward cascade would be limited by low frequency CTB performance.

More data is required to define CTB distortion phase at various frequencies so that a system design rule can be generated.


1 Bell Telephone Laboratories. "Transmission Systems for Communications", 4th Edition, Western Electric Co. Ltd. Technical Publications, 1971.

2 Bell, Richard; and Rebeles, P. "Third Order Distortion Build Up In A Multi-Channel Cascade", NCTA Convention Papers, 1973.

3 Prochazka, A. "Improving CATV System Performance Through Amplifier Design", CCTA Convention Papers, 1975.

4 Bauer, Dieter; and Reichert, Harry. "400 MHz CATV System Performance Studies", NCTA Convention Papers, 1981.

5 Cowen, Marty - Scientific-Atlanta correspondence.

6 Do, Steven - Motorola correspondence.

7 McEwen, Doug; Slater, Norm; Straus, Tom; Pidgeon, Rezin. "Distortion Characteristics of Integrated Fibre, AML and Cable Amplifier Systems", CCTA Convention Papers, 1990.

## Doug McEwen, P.Eng.

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Prior to joining CUC in 1986, Doug worked in the Network Development group of Cablesystems Engineering with Rogers. His responsibilities there included network design and calculating the technical performance and cost of various design options.

Doug received his Bachelor of Engineering Science from the University of Western Ontario in London in 1983. He is a member of the Association of Professional Engineers of Ontario.

## Rezin Pidgeon

Rezin Pidgeon is a Principal Engineer for Scientific-Atlanta and currently supports the fiber optic development group. Rezin has been with Scientific-Atlanta since 1962, and was with the Antenna and Telecommunications Instrument groups until 1980. Rezin has been with the Broadband Communications Business Division since then.

Prior to joining Scientific-Atlanta, Rezin was a senior engineer at the Georgia Tech Engineering Experiment Station. Rezin holds BSEE and MSEE degrees from Georgia Institute of Technology, and is a member of I.E.E.E., the Society of Cable Television Engineers, and ETA Kappu Nu.

## Norm Slater, P.Eng.

Norm joined Comlink Systems in January 1986 as Manager of CATV Systems Engineering.

Educated at the University of Western Ontario, Norm received his B.E.Sc. degree, Electrical Engineering in 1975 and also the Harry Cross Medal for Electrical Engineering.

Commencing in 1975 up to the end of 1985, Norm was employed by Cablesystems Engineering latterly as Manager, Network Engineering where his responsibilities included the development and implementing of network designs for all Rogers' systems.

Norm has submitted several technical papers to the Canadian Cable Television Association and the U.S. National Cable Television Association. He is a member of the Association of Professional Engineers of the Province of Ontario.