CONVERTER NOISE MEASUREMENT DEFINITIONS AND CONVERSION DERIVATIONS

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ABSTRACT

When evaluating noise introduced by cable television converters, terms such as carrier-to-noise ratio, signal-to-noise ratio, and noise figure are commonly used. These terms are measures of noise performance. The ability to use these terms interchangeably has become increasingly important with the advent of baseband converters. Baseband converters have precluded the use of traditionally used noise figure equipment which is widely used for evaluating RF converters. Therefore, a complete understanding of noise terms is essential. This paper will identify the differences between the commonly used noise terms along with the derivation of the conversion factors.

INTRODUCTION

For years, noise figure has been the unwritten standard for specifying the noise performance of RF cable television converters. However, the substantially different architecture of a baseband converter precludes the use of a noise figure meter for measuring noise performance of baseband converters. One method used for measuring noise performance in a baseband converter is illustrated in figure 1. In this configuration noise performance is measured in the form of signal-to-noise ratio of the baseband video signal.

Even though video S/N ratio represents the true picture quality delivered to a customer's television set, it does not clearly indicate the level of noise the converter adds to the transmitted signal. Noise figure is a more familiar way to represent noise added by a converter. Fortunately, through mathematical manipulation, there is a conversion factor that can be added to the unweighted S/N measurement for translating video S/N to noise figure. The following equation shows this conversion,

$$NF = C_p(in)(dBmV) + 59.21(dBmV) - S/N(dB) - 6.86(dB) \tag{1}$$

where:

$C_p(in)$ = input carrier power level into the converter

S/N = unweighted signal-to-noise ratio of baseband video signal

Noise BW = 4.0 MHz

Reference Impedance = 75Ω

To help understand this equation, the mathematical derivation for translating video signal-to-noise to noise figure will be presented. Also, several terms that are commonly used when describing noise performance will be defined.

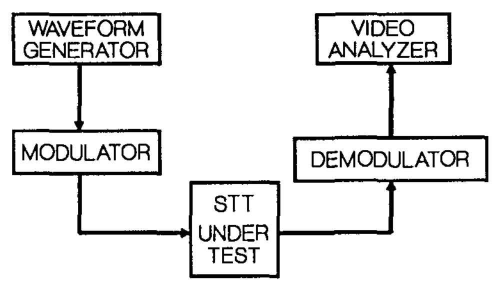

Fig. 1 Typical Signal-To-Noise Test Configuration

DEFINITION OF NOISE TERMS

When measuring noise performance in CATV systems, several terms are used to define the relative amounts of noise produced. The most common terms are: signal-to-noise ratio (S/N), carrier-to-noise ratio (C/N), thermal noise, noise figure, noise equivalent bandwidth, and weighting filters. The following is a brief definition of these commonly used noise terms.

Signal-to-Noise Ratio

Signal-to-noise ratio as defined by the International Radio Consultative Committee (CCIR) is a measurement of noise performance made on a baseband video signal. CCIR defines S/N as the difference in voltage between blanking level and peak white to the RMS noise voltage weighted by a prescribed noise filter.

$$\left[\frac{Sv}{Nv}\right]_{CCIR} = \frac{\text{Blanking to peak white}}{\text{(RMS) Noise voltage in a reference bandwidth}} \tag{2}$$

In decibels,

$$\frac{Sv}{Nv}(dB) = 20 \cdot Log \left[\frac{Sv}{Nv}\right] \tag{3}$$

It should be noted here that other institutions such as EIA and Bell Telephone Laboratories (BTL) define signal-to-noise ratio differently than CCIR. However, for this discussion only the CCIR's definition of S/N ratio will be presented.

Carrier-to-Noise Ratio

Carrier-to-noise ratio as defined by the National Cable Television Association (NCTA) is the power of the carrier signal during the sync pulse to the noise power in a bandwidth of 4 MHz. This bandwidth is used because it is approximately the bandwidth of the baseband video signal.

$$\left[\frac{Cp}{Np}\right]_{NCTA} = \frac{\text{Carrier power during sync pulse}}{\text{Total noise power in a reference bandwidth}} \tag{4}$$

In decibels,

$$\frac{Cp}{Np}(dB) = 10 \cdot Log \left[\frac{Cp}{Np}\right] \tag{5}$$

Similar to the S/N definition, the Television Allocation Study Organization (TASO) defines C/N differently than the NCTA. However, for this discussion only the NCTA's definition for C/N ratio will be discussed.

Thermal Noise

Thermal noise is due to the random fluctuation of electrons in any conducting medium whose temperature is above absolute zero. Thermal noise is also referred to as white noise because it has been shown experimentally and theoretically to have a uniform spectrum up to frequencies on the order of 10^{13} Hz. In a CATV system, the level of thermal noise with respect to the signal or carrier determines the amount of visible snow viewed on a television receiver. As the definition implies, a resistor is a thermal noise source, with the level of noise being dependent on the physical temperature of the resistor. By definition of noise figure a reference temperature of 290°K will be used for determining the level of thermal noise generated by a resistor. The mean squared noise voltage at the output of a resistor R can be shown to be,

$$n_e^2 = 4 \cdot R \cdot B \cdot T \cdot K \tag{6}$$

where:

- n_e = the RMS noise voltage
- R = resistance of circuit
- T = temperature in Kelvin, (°K = °C + 273.15)
- B = bandwidth of measurement system, 4 MHz is the bandwidth of most video systems
- k = Boltzman constant (1.38 x 10^{-23} watts/°K)

By the maximum power theorem, the maximum available noise power from the resistor is given by,

$$n_p = \frac{n_e^2}{4 \cdot R} \quad \text{or} \quad n_p = K \cdot T \cdot B \tag{7}$$

Consider the following example shown in figure 2, where the resistance of R is equal to 75Ω and the temperature is equal to 290°K. After substituting the appropriate numbers into equation 6 the mean squared voltage is equal to 4.802×10^{-12} Volts^2, or

$$n_e^2 = 4 \cdot 75\Omega \cdot 4\mathrm{MHz} \cdot 1.38^{-23}(\mathrm{watts}/°\mathrm{K}) \cdot 290°\mathrm{K} \tag{8}$$

After applying the maximum power transfer theorem to the 4.802×10^{-12} Volts^2 noise source and converting to dB referenced to a millivolt (dBmV) the available noise power from the resistor equals -59.21 dBmV.

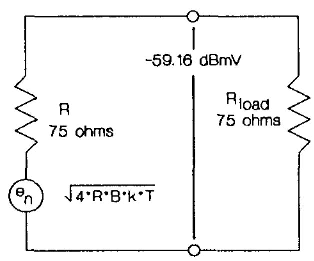

Fig 2. 75Ω Thermal Noise Source

Noise Figure

The noise figure (NF) of a two-port network gives a measure of the degradation of the S/N or C/N between the input and output ports. Noise figure compares the ratio of the carrier power to noise power at the input, to the carrier-to-noise power at the output, or expressed in dB

$$NF = 10\mathrm{Log}\left[\frac{\frac{Cp(in)}{Np(in)}}{\frac{Cp(out)}{Np(out)}}\right] \tag{9}$$

By definition, the input noise power, $N_p(in)$, in equation 9 is equivalent to the thermal noise power generated by a resistor matched to the input and at a temperature of 290°K. In other words the input noise power is equal to, $K \cdot 290 \cdot B$. The noise figure of an ideal device be it a television converter, amplifier, or other circuit would equal 0 dB. As the converter or device introduces unwanted noise the noise figure will increase to some power ratio greater than 0 dB.

Consider a cable television converter that has a noise figure of 12dB. A 12dB noise figure will increase the output noise floor by 12dB over the input thermal noise floor. In other words there will be a 12dB degradation of the noise performance at the output of the measured converter.

Equivalent Noise Bandwidth

When flat broadband noise is transmitted through a communication system, the total noise power at the output becomes a function of the bandwidth and shape of the system's transfer function. Consider a filter whose transfer function can be related by the mean square voltage A_v^2. The frequency response of this filter is shown by the solid line in figure 3. If the input to this filter is white noise with mean-square voltage V_{in}^2/Hz, the corresponding output voltage in a 1-hertz interval at frequency f is,

$$\frac{V_{out}^2}{\mathrm{Hz}} = \left|A_v(f)\right|^2 \cdot V_{in}^2 \tag{10}$$

Integration over the entire frequency band yields,

$$\int_0^\infty V_{out}^2(f)\,df = V_{in}^2 \cdot \int_0^\infty \left[\left|A_v(f)\right|\right]^2 df \tag{11}$$

The dashed line in figure 3 represents the equivalent noise bandwidth resulting from the integration. Basically, the equivalent noise bandwidth is the value that gives equal areas under the solid and dashed curves such that,

$$B = \frac{\int_0^\infty \left[\left|A_v(f)\right|\right]^2 df}{\left[\left|A_m\right|\right]^2} \tag{12}$$

Understanding equivalent noise bandwidth is essential when taking into account vestigial filters and baseband weighting filters in the noise analysis of a television system. Consider the unified baseband weighting filter shown in figure 5. The equivalent noise bandwidth of this filter integrated from 0 Hz to 4.2 MHz is equal to 0.881 MHz. This filter will reduce the level of broadband thermal noise power, which is flat vs. frequency in a 4.2 MHz video bandwidth by 6.8 dB.

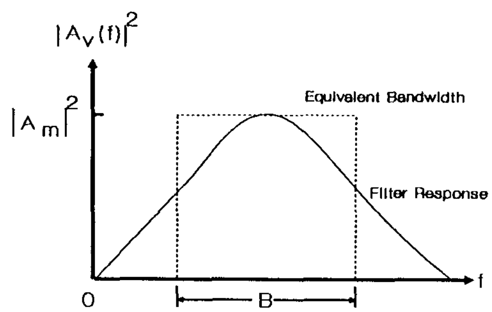

Fig. 3 Filter and Noise Equivalent Bandwidth

Weighting Filters

Studies have shown that the perceptibility of noise in television pictures varies with frequency. Noise at high frequencies is less noticeable to the viewer than noise at low frequencies. Weighting filters are intended to reduce the noise at higher frequencies in the same proportion as the noise perception to the viewer is reduced. The shape was determined by introducing controlled amounts (at different frequency bands) of noise into television pictures and having a group of viewers rate degradation.

Several institutions such EIA and CCIR have adopted weighting filters. The most commonly used weighting are those adopted by CCIR. The weighting filter of CCIR recommendation 423-1 is shown in figure 4. This was developed for System M (NTSC). Other weighting have been developed for other systems such as PAL. More recently, a weighting filter referred to as the CCIR Unified was adopted which is somewhat of a compromise between the original NTSC weighting and that of other systems. This weighting is shown in figure 5. It is intended for use in both NTSC and other systems. All of these comments apply to the luminance signal and do not consider chrominance noise.

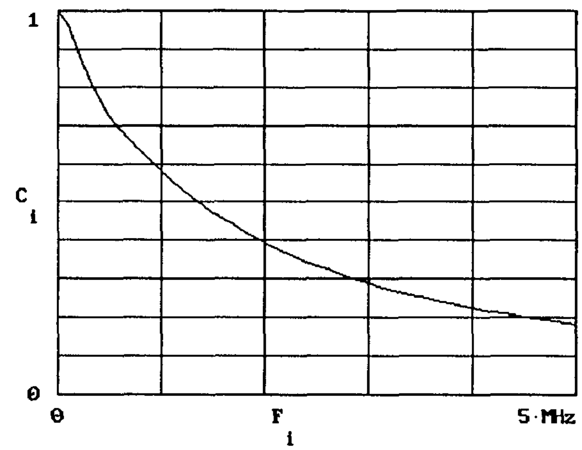

Fig 4. Weighting Filter, CCIR 423-1

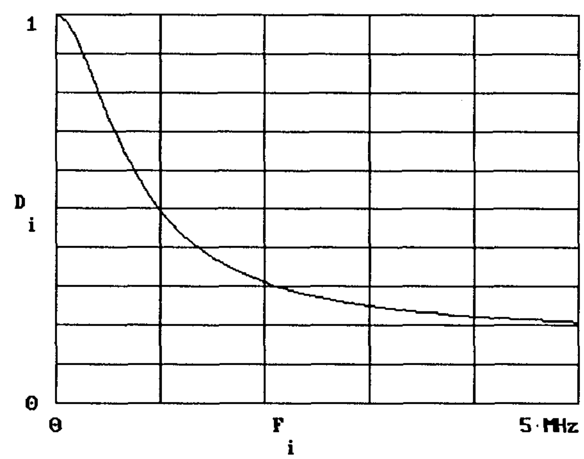

Fig 5. Weighting Filter, Unified

SIGNAL-TO-NOISE TO NOISE FIGURE DERIVATION

Noise Figure Expanded

It was previously shown that the noise figure of an amplifier is a measure of the ratio of carrier power to noise power at the input, to the carrier-to-noise power at the output. In terms of voltage in dB, noise figure can also be expressed as,

$$NF = 20 \cdot \text{Log}\left[\frac{\dfrac{Cv(in)}{Nv(in)}}{\dfrac{Cv(out)}{Nv(out)}}\right] \tag{13}$$

After applying some well know logarithmic identities, NF can be expanded as follows,

$$NF = 20 \cdot \text{Log } C_V(in) - 20 \cdot \text{Log } N_V(in) - 20 \cdot \text{Log}\left(\frac{C_V(out)}{N_V(out)}\right) \quad (14)$$

The term $20 \cdot \text{Log}C_V(in)$ represents the input signal level in dBmV, and $20 \cdot \text{Log}N_V(in)$ represents the input noise voltage in dBmV. Equation 14 is valid only if the input noise is exclusively due to thermal noise at 290°K. Since we are considering a 75Ω system, the thermal noise power deliverable by a 75Ω resistor at 290°K is equal to -59.21 dBmV. After replacing 20 Log $N_V(in)$ in equation 14 with -59.21 dBmV, NF becomes,

$$NF = 20 \cdot \text{Log } C_V(in) + 59.21 \text{ (dBmV)} - 20 \cdot \text{Log}\left(\frac{C_V(out)}{N_V(out)}\right) \quad (15)$$

The third term in equation 15, $20 \cdot \text{Log}\{Cv(out)/Nv(out)\}$, represents the output C/N ratio of the converter under test. However, when analyzing baseband television converters the output C/N ratio is not readily available. Again with a little mathematical manipulation C/N ratio can be expressed in terms of S/N ratio.

S/N to C/N Relationship

The relationship between RF carrier-to-noise ratio and baseband video signal-to-noise ratio can be shown by expanding and rearranging terms in the detected AM wave equation. Before expanding the detected wave equation lets first consider a carrier that has been amplitude modulated by a baseband video signal. A simplified expression showing the time domain representation of a video modulated RF carrier is shown as,

$$V_{AM} = V_c(1 - D + D \cdot \text{Cos } w_m t)\text{cos}w_c t \quad (16)$$

where:

V_c = amplitude of carrier
w_c = frequency of carrier
w_m = frequency of video signal
D = modulation ratio

The modulation ratio for a video modulated signal expressed in percent, is referred to as depth of modulation (DOM). DOM is the difference between the maximum and the minimum level of the RF envelope amplitude expressed as a percentage of the maximum RF envelope level, or

$$DOM = \frac{A - B}{A} \cdot 100 \quad \% \quad (17)$$

Where A in figure 6 is equal to the peak-to-peak amplitude of the RF carrier and B is equal to the peak-to-peak amplitude of the RF carrier during white level modulation. Therefore, the modulation ratio, D, in equation 16 is simply equal to:

$$D = \frac{A - B}{2 \cdot A} \quad (18)$$

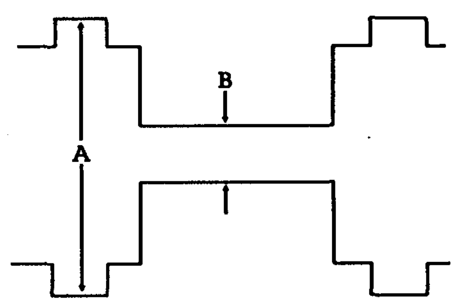

Fig 6. Video Depth Of Modulation

The predominate noise in most RF systems is usually generated at RF. The sidebands of this broadband noise with respect to the carrier are uncorrelated. The uncorrelated noise voltage sidebands will add in a RMS fashion upon detection. The following equation shows the AM wave equation with constant amplitude broadband noise added:

$$V_{AM} = V_c(1 - D + D \cdot \text{Cos } w_m t)\text{Cos } w_c t + V_n \cdot \text{Cos}[w_n t + \alpha(t)] \quad (19)$$

Where, V_n is the noise voltage and $\alpha(t)$ is the phase angle of the noise which varies randomly from 0 to 2π. This noise model expresses noise voltage in a narrow band of fixed sinusoidal amplitudes but variable phase.

Now consider synchronously detecting the video information and noise in equation 19. This is performed by multiplying equation 19 by a frequency and phase coherent signal equal to Cos w_c, or

$$V_{det} = \{V_c(1 - D + D \cdot \text{Cos } w_m t)\text{Cos } w_c t + V_n \cdot \text{Cos}[w_n t + \alpha(t)]\} V_o \cdot \text{Cos } w_c \tag{20}$$

After low pass filtering, V_{det} becomes

$$V_{det} = \frac{V_c V_o}{2}(1 - D) + \frac{V_c V_o D}{2} \cdot \text{Cos } w_m t + \frac{V_n V_o}{2} \cdot \text{Cos}((w_c - w_n)t + \alpha(t)) \tag{21}$$

The first term in equation 21 represents a DC term which can be ignored in the S/N to C/N translation. The second term represents the detected peak signal voltage. At peak modulation when Cos $w_m t$ is equal to negative one, the detected peak-to-peak signal can be shown as,

$$S_v = V_c \cdot V_o \cdot D \tag{22}$$

Since we are dealing with video modulation the detected peak-to-peak signal during peak white modulation is defined from sync tip to peak white. Recall that when expressing S/N ratio, CCIR defines signal from blanking to peak white. The CCIR signal level correction is made by multiplying the detected peak-to-peak signal, sync tip to peak white, by the ratio of blanking level to peak white with respect to sync tip to peak white. The following equation shows the correction factor ratio,

$$A = \frac{\text{Peak White - Blanking}}{\text{Peak White - Sync Tip}} \tag{23}$$

After correcting for CCIR's definition of signal, the detected signal can be expressed as:

$$S_v = V_c \cdot V_o \cdot D \cdot A \tag{24}$$

The final term in equation 21 is the detected peak noise voltage. The detected peak noise can be expressed in RMS noise density as,

$$Nvd = \frac{Vn \cdot Vo}{2 \cdot \sqrt{2}} \qquad \frac{Volts}{Hz} \tag{25}$$

Now that we have an expression for noise density, the total noise voltage can be determined by integrating the noise voltage over the appropriate RF bandwidth times the baseband response. In the case of a baseband converter the equivalent RF bandwidth will be determined by the Nyquist filter response shown in figure 7. Where as the baseband response will be determined by bandlimiting filters or weighting filters. For now we will presume an ideally flat baseband frequency response without any bandlimiting or weighting filters. Since the uncorrelated noise sidebands add in a RMS fashion, the equivalent noise bandwidth is determined by the following integration:

$$B_{NYQ} = \sqrt{\int_0^{f_h} NYQ(f)^2 \, df - \int_0^{-f_n} NYQ(f)^2 \, df} \tag{26}$$

Where:

f = the frequency offset from the carrier

NYQ = the Nyquist filter voltage frequency response

f_n = lower Nyquist filter cutoff frequency

f_h = upper Nyquist filter response

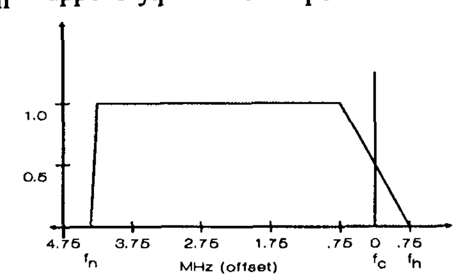

Fig 7. NTSC Nyquist Filter Response

By knowing the equivalent noise bandwidth, the total RMS noise voltage can be expressed as:

$$N_v = \frac{V_n \cdot V_o}{2\sqrt{2}} \cdot B_{NYQ} \tag{27}$$

After reducing the carrier voltage, V_c, by a factor of two, as a result of the Nyquist slope, the detected signal to noise voltage can be expressed as:

$$\frac{Sv}{Nv} = \frac{\dfrac{Vc \cdot Vo \cdot D \cdot A}{2}}{\dfrac{Vn \cdot Vo \cdot B_{NYQ}}{2 \cdot \sqrt{2}}} \tag{28}$$

Simplifying,

$$\frac{Sv}{Nv} = \frac{Vc}{Vn} \cdot \frac{2 \cdot \sqrt{2}}{2} \cdot (D \cdot A) \cdot \frac{1}{B_{NYQ}} \tag{29}$$

Notice that the first term in equation 29 represents the ratio of the carrier voltage to noise density. Since we need to derive an expression for carrier to noise power in a reference bandwidth, the noise density needs to be multiplied by a reference bandwidth, B_{ref}. Without disturbing the integrity of equation 29, S/N voltage can be rewritten as:

$$\frac{Sv}{Nv} = \frac{Vc}{Vn \cdot \sqrt{B_{ref}}} \cdot \frac{\sqrt{2}}{2} \cdot (2 \cdot D \cdot A) \cdot \frac{\sqrt{B_{ref}}}{B_{NYQ}} \tag{30}$$

Expressing S/N voltage as S/N power in dB, equation 30 can be rewritten as:

$$\frac{Sp}{Np} = 20 \cdot \mathrm{Log}\left[\frac{Vc}{Vn \cdot \sqrt{B_{ref}}} \cdot \frac{\sqrt{2}}{2} \cdot (2 \cdot D \cdot A) \cdot \frac{\sqrt{B_{ref}}}{B_{NYQ}}\right] \tag{31}$$

Notice that the first term in equation 31, $V_c/(V_n\sqrt{B_{ref}})$, represents the carrier-to-noise ratio as defined by the NCTA.

<u>S/N to Noise Figure Relationship</u>

As previously shown noise figure can be expressed as,

$$NF = 20 \cdot \mathrm{Log}\, C_v(in) + 59.21\ (dBmV) - 20 \cdot \mathrm{Log}\left(\frac{C_v(out)}{N_v(out)}\right) \tag{32}$$

Also recall that the expression $20 \cdot \mathrm{Log}\{Cv(out)/Nv(out)\}$ needs to be expressed in terms of S/N ratio. Therefore, after solving for C/N in equation 31, C/N can be expressed as

$$\frac{Cp}{Np} = \frac{Sp}{Np} - 20 \cdot \mathrm{Log}\left[\frac{\sqrt{2}}{2}\right] - 20 \cdot \mathrm{Log}(2 \cdot D \cdot A) - 20 \cdot \mathrm{Log}\left[\frac{\sqrt{B_{ref}}}{B_{NYQ}}\right] \tag{33}$$

Substituting C/N in equation 32,

$$NF = 20 \cdot \mathrm{Log}Cv(in) + 59.21\ (dBmV) - \frac{Sp}{Np} + 20 \cdot \log\left[\frac{\sqrt{2}}{2}\right] + 20 \cdot \mathrm{Log}(2 \cdot D \cdot A) + 20 \cdot \mathrm{Log}\left[\frac{\sqrt{B_{ref}}}{B_{NYQ}}\right] \tag{34}$$

Now let's consider a detected NTSC video signal where the modulation ratio, D, is equal to .4375 and the CCIR correction factor, A, is equal to .7142. The noise power will be measured in a reference bandwidth of 4 MHz and a Nyquist equivalent bandwidth of 3.8 MHz. The noise figure of this signal can be shown as:

$$NF = 20 \cdot \text{Log}\, C_V(\text{in}) + 59.21\ (\text{dBmV}) - \frac{S_p}{N_p} - 3\ (\text{dB}) - 4.08\ (\text{dB}) + .222 \qquad (35)$$

Simplifying the noise figure expression:

$$NF = C_p(\text{in})(\text{dBmV}) + 59.21(\text{dBmV}) - S/N(\text{dB}) - 6.86(\text{dB}) \qquad (36)$$

where:

$C_p(\text{in})$ = input carrier power level into the converter
S/N = unweighted signal-to-noise ratio of baseband video signal
Noise BW = 4.0 MHz
Reference Impedance = 75Ω

Conclusion

It has been shown that noise figure can be determined from the baseband signal-to-noise ratio measurement. However, the reader must be cautioned that the numbers used in the S/N ratio to noise figure translation are dependent on several factors. Factors such as depth of modulation, Nyquist slope, reference bandwidth, and the level of thermal noise will influence the accuracy of the conversion factors. Care must be taken when measuring baseband signal-to-noise in order to have reliable and repeatable results.

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