THE EFFECTS OF OSCILLATOR PHASE NOISE IN AN FM VIDEO LINK

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ABSTRACT

Phase noise of local oscillators in frequency converters in an FM Video link can introduce noise into TV pictures in a CATV system. The acceptable level of such noise have not been established. The qualitative measure of phase noise is its perceptibility in a TV picture. Baseband signal-to-noise ratio (SNR) is a quantitative measure. This paper presents the theory relating the RF noise spectrum of an oscillator and the baseband noise spectrum and SNR for an FM Video link. Measured results are compared to the theory and conclusions about perceptibility presented.

INTRODUCTION

The NCTA Engineering committee formed its HDTV Subcommittee in the Fall of 1987. Group 1 of that subcommittee is charged with the investigation and documentation of signal transfer characteristics in cable systems, with particular emphasis on parameters useful in forecasting the transparency of a cable system to various HDTV proposals. Improved quality of present CATV service is also an expected result. This paper, together with the companion paper by Rezin Pidgeon [1] form the first published results of the group 1 investigations.

It was determined that the first efforts of the Group should be devoted to rigorous investigations of phase noise effects throughout the entire network, from satellite link through final detection. This paper treats the effects of phase noise in NTSC FM Video links used in satellite transmission of CATV signals. Both RF and baseband theoretical and measued results are presented.

THEORY

FM System Equations

FM modulation is described by the following equation [1]:

$$M(t) = A_c \cdot \cos\left[2 \cdot \pi \cdot f_c \, t + X \cos\left[2 \cdot \pi \cdot f_m \, t\right]\right] \qquad \text{EQ 1}$$

where $M(t)$ = the modulated waveform
X = the modulation index
= peak phase deviation
f_m = modulating frequency

and

$$X = \frac{\Delta f_m}{f_m} \qquad \text{EQ 2}$$

where Δf_m = peak frequency deviation

In the general case, this implies a Bessel series of sinusoids (sidebands) when expanded. When X is small (X 0.5 rad), the process approaches linearity and the series is adequately approximated by the first term. The result is a single pair of sidebands offset to either side of the carrier by wm. By constructing a phasor diagram of the carrier and sidebands one can determine that:

$$A_s = \frac{1}{2} \text{ARCTAN}(X) = \frac{X}{2}$$

where V_s = peak voltage amplitude of sideband

Noise can generally be treated with this approximation unless the total noise power is great enough to produce large peak phase deviation. Low frequency noise components may also violate the approximation since phase deviation due to a given frequency deviation increases as modulating frequency decreases (see EQ 2).

Thermal Noise

Consider a carrier with flat thermal noise input to an FM demodulator. By "thermal noise" we mean noise with the statistical properties of that generated by resistors due to thermal effects. The actual source of this noise in an FM video link would include galactic and ground noise entering the antenna, amplifier gain and internal noise etc. It does not include oscillator phase noise. The following parameters will be used for the predetection signal:

C = carrier power (Watts)
N_t = thermal noise power density (Watts/Hz)
V_c = peak carrier voltage (Volts)

The signal at baseband will be:

$$V_s = k_d f_s = k_d f_p \frac{1}{1.4} \qquad \text{EQ 3}$$

where V_s = peak to peak signal voltage
K_d = demodulator constant (V 2πsec/rad)
Δf_p = peak frequency deviation due to 140 IRE total video

Our objective is to calculate video signal to noise which in NCT 7 is defined as the ratio of peak to peak signal volts of 100 IRE active video to RMS noise voltage (for noise above 10 KHz) [3]. The RMS noise voltage at baseband (out of the demodulator) is given by:

$$V_{bbn0} = k_d \Delta f_n \qquad \text{EQ 4}$$

where Δf_n = RMS deviation due to noise input to demodulator

The phase deviation produced by a single 1 Hz wide band of noise offset from the carrier by f_m is:

$$\Delta\phi_{NSSB} = \frac{\text{RMS NOISE VOLTAGE}}{V_c} = \frac{1}{\sqrt{2}}\sqrt{\frac{N_t}{C}}$$

There are two such bands of noise, one above and one below the carrier frequency. The noise in these bands is uncorrelated, and therefore additive in power. The RMS phase deviation due to the combination of both is:

$$\Delta\phi_N = \frac{1}{\sqrt{2}}\sqrt{\frac{2N_t}{C}} = \sqrt{\frac{N_t}{C}} \qquad \text{EQ 5}$$

Referring to EQ 2, 4 & 5, we have:

$$V_{BBNO}\left[f_m\right] = k_d f_m \sqrt{\frac{N_t}{C}} \qquad \text{EQ 6}$$

This is the RMS noise voltage at baseband frequency f_m at the output of the demodulator due to the thermal noise accompanying the carrier at the input. The total noise voltage may be found by integration over the desired baseband frequency range. The voltages at various frequencies are uncorrelated requiring integration of power (voltage squared).

$$V_{BBN} = \int_0^B \left[V_{BBNO}\right]^2 df \qquad \text{EQ 7}$$

$$V_{BBN} = k_d \frac{B^{\frac{3}{2}}}{\sqrt{3}}\sqrt{\frac{N_t}{C}} \qquad \text{EQ 8}$$

Using the previous definition of signal to noise and EQ 3 & 8:

$$\frac{S}{N_t} = \frac{f_s\sqrt{12}}{B^{\frac{3}{2}}}\sqrt{\frac{C}{N_t}} \qquad \text{EQ 9}$$

This gives the signal to noise without pre/deemphasis or noise weighting. It includes all baseband frequencies. In an NTSC FM video link, preemphasis is used to condition the signal for the FM format. After demodulation, the signal is deemphasized to restore the original baseband signal. Preemphasis for NTSC video attenuates low frequencies by 10 dB and increases high frequencies by about 3 dB. This reduces variation of average carrier frequency due to DC variation in the video and improves signal to noise. The latter effect can be seen by referring to EQ 4 and noting that baseband noise voltage due to thermal noise increases linearly with frequency, giving a triangular baseband noise spectrum. The deemphasis will have 10 dB of gain at low frequencies and 3 dB attenuation at high frequencies. Thus the noise is boosted at low frequencies where it is small and attenuated at high frequencies where it is large. The deemphasis function according to CCIR Recommendation 405-1 is:

$$D(f) = 10 \cdot \frac{1 + B \cdot f^{2}}{1 + C \cdot f^{2}} \qquad \text{EQ 10}$$

where D(f)= The deemphasis power response function

$$B = 1.306 \cdot 10^{-12}$$

$$C = 28.58 \cdot 10^{-12}$$

The important factor in video noise is how it is perceived by the viewer. Early studies showed that the perceptual effects of noise are very frequency dependent. Generally, the noise is perceived less as frequency increases. Weighting filters have been devised to have a response in frequency which matches the perceptual response of the human visual system. The weighting filter given in CCIR Report 637-1, equation 4 "for system M (prior to the introduction of the unified network)" is :

$$W(f) = \frac{1 + \left[\frac{f}{f3}\right]^{2}}{\left[1 + \left[\frac{f}{f1}\right]^{2}\right]\left[1 + \left[\frac{f}{f2}\right]^{2}\right]} \qquad \text{EQ 11}$$

where W = The weighting power response function

f1 = 0.270 MHz
f2 = 1.37 MHz
f3 = 0.390 MHz

This network is in general use for NTSC system measurements. Taking EQ 3, 6 & 7 and including the power response of the above deemphasis and weighting functions for noise above 10 KHz gives:

$$\left[\frac{S}{N_t}\right] = \frac{2 \cdot f_p \cdot \frac{1}{1.4} \cdot \sqrt{\frac{C}{N_t}}}{\sqrt{\int_{10 \cdot \mathrm{KHz}}^{4.2 \cdot \mathrm{MHz}} H(f) \cdot f^{2}\, df}} \qquad \text{EQ 12}$$

where H(f) = D(f) W(f)

Carrying out the integration gives a constant so that:

$$K.t = \sqrt{\int_{10 \cdot \mathrm{KHz}}^{4.2 \cdot \mathrm{MHz}} H(f) \cdot f^{2}\, df} \qquad \text{EQ 13}$$

$$= 1.144 \cdot 10^{9}$$

The units of K_t are the square root of Hz cubed. In all equations frequencies are in Hz and power densities are in 1Hz bandwidth. Change of units can be confusing.

Phase Noise

Now, let us consider phase noise which might be introduced by a local oscillator or carrier generator in the system. Oscillator phase noise is caused by noise voltages in the oscillator tuning circuit which produce frequency variation. These effects are described in some detail in a companion paper [1]. We will consider the phase noise to have a power density spectrum which is proportional to 1/fm^2. Thus the noise density decreases at 6 dB per octave as offset frequency increases. This is true of free running oscillators. Where synthesizers are used, the noise within the control bandwidth is a function of their design. While we will not treat that case here, the information presented is readily applied.

Unlike thermal noise, the phase noise sidebands are correlated since both arise from the same source. We add one new parameter to describe the signal at the demodulator input:

N_p = phase noise power density

Which is exactly that described above. The analysis proceeds as for thermal noise save two points. First, the noise power density is not flat and second, the sidebands are correlated. In this paper I will account for the frequency dependence through use of a reference offset. That is, I will specify phase noise power density at a particular offset frequency. By this means, the carrier to phase noise density can be written as:

$$\frac{C}{N_p} = \left[\frac{C}{N_{pr}}\right] \cdot \left[\frac{f_m}{f_r}\right]^2 \qquad \text{EQ 14}$$

where N_{pr} = phase noise power density at offset fr

or using the notation of REF 2:

$$Q_e = \sqrt{\frac{C}{N_{pr}}} \cdot \frac{1}{f_r}$$

We can now write RMS phase deviation for one sideband as:

$$\Delta\phi_{NSSB} = \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{N_p}{C}}$$

Since the sidebands are correlated, the two add as <u>voltage</u> which gives:

$$\Delta\phi_n = \frac{2}{\sqrt{2}} \cdot \sqrt{\frac{N_p}{C}} \quad \phi_n = \sqrt{2} \cdot \frac{f_r}{f_m} \cdot \sqrt{\frac{N_{pr}}{C}} \qquad \text{EQ 15}$$

Referring to EQ 2, 4 & 15, the baseband noise voltage is given by:

$$V_{BBNO}(f) = k_d \cdot \sqrt{2} \cdot f_r \cdot \sqrt{\frac{N_{pr}}{C}} \qquad \text{EQ 16}$$

We see from this that the <u>baseband</u> spectrum due to phase noise is flat. You will recall that the RF phase noise spectrum (predetection) rolled off at 6dB/octave. Taking EQ 3, 7 & 16 and including deemphasis and weighting we can calculate signal to noise:

$$\left[\frac{S}{N_p}\right] = 20 \cdot \mathrm{LOG} \cdot \left[\frac{2 \cdot f_p \cdot \frac{1}{1.4} \cdot \sqrt{\frac{C}{N_{pr}}}}{\sqrt{2} \cdot f_r \cdot \sqrt{\int_{10 \cdot \mathrm{KHz}}^{4.2 \cdot \mathrm{MHz}} H(f)\, df}}\right]$$

Carrying out the integration yields:

$$K_p = \sqrt{\int_{10 \cdot \mathrm{KHz}}^{4.2 \cdot \mathrm{MHz}} H(f)\, df} = 1.545 \cdot 10^3$$

We now have a complete picture of both the effects of thermal and phase noise in an FM video link. The effects are readily combined. Each mechanism is independent and the baseband noise arising from each is uncorrelated. The noise power from the individual effects may be added. The video signal to noise ratio is defined in terms of voltage. From these facts we can determine the overall video signal to noise in terms of the thermal and phase noise. Thus:

$$\left[\frac{S}{N}\right] = 20 \cdot \mathrm{LOG} \cdot \sqrt{\frac{1}{\left[\frac{N_t}{S}\right]^2 + \left[\frac{N_p}{S}\right]^2}} \qquad \text{EQ 19}$$

Take as an example the case where:

(C/N_t)dB = 86 dB
(C/N_p)dB = 66 dB
f_r = 20 KHz

then

(S/N_t)dB = 48.6 dB
(S/N_p)dB = 56.9 dB

and

(S/N)dB = 48.0 dB

Summary of Theoretical Results

Fig 1 shows the predetection spectrum using 1 KHz bandwidth and the parameters of the above example. These are computed plots. Actual analyzer display of noise is 1.7 dB less than the true value [4]. The figure shows the thermal and phase spectra separately and the overall spectrum. The rolloff of phase noise and flat thermal noise of the predetection spectrum are evident.

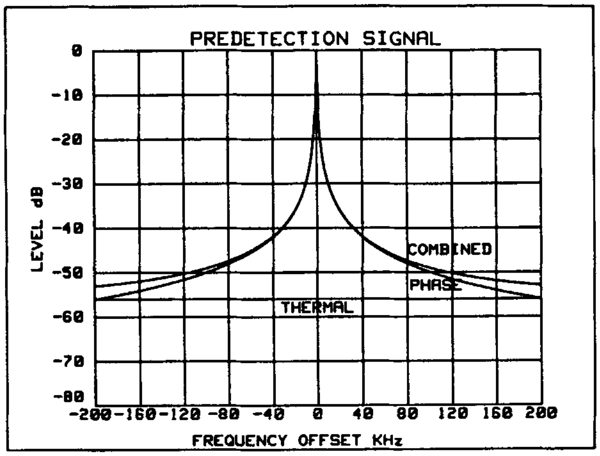

FIGURE 1

Fig 2 shows the baseband spectrum for the same case without deemphasis or weighting. Here, we see the flat baseband spectrum due to phase noise and the rising spectrum of the thermal noise. The thermal noise is dominant except for very low frequencies.

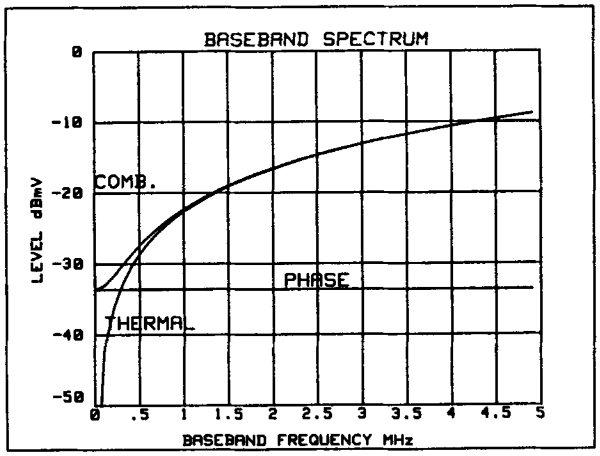

FIGURE 2

Fig 3 shows the same baseband spectra with the added effects of deemphasis and weighting. The relationship of the two types of noise in this plot is representative of the perceived effects. The boost of 10 dB by deemphasis is evident in the low frequencies. Clearly, phase noise dominates the low frequency area. However, the phase noise in this example degrades the overall S/N very little (see above example)

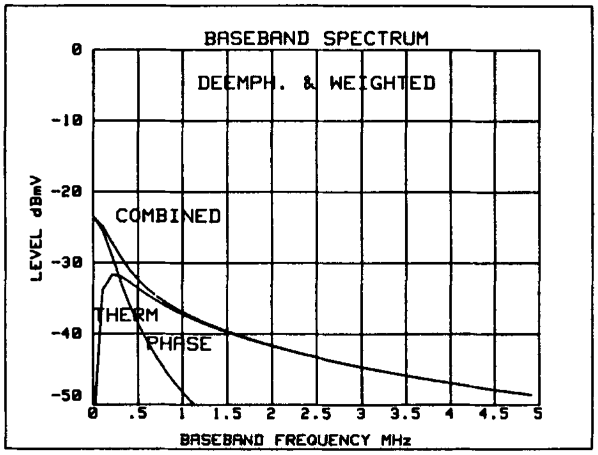

FIGURE 3

Fig 4 presents a family of curves for various thermal and phase noise conditions. All phase noise is referenced at 20 KHz offset and all densities are per Hz. These allow ready determination of system signal to noise for most combinations of thermal and phase noise. With a spectrum analyzer it is not practical to display power density in 1 Hz bandwidth. Other bandwidths may be used by subtracting 10 LOG (BW) from the thermal and phase noise scales, where BW is the desired bandwidth in Hz. For example, Fig 5 is referenced to 1 KHz BW.

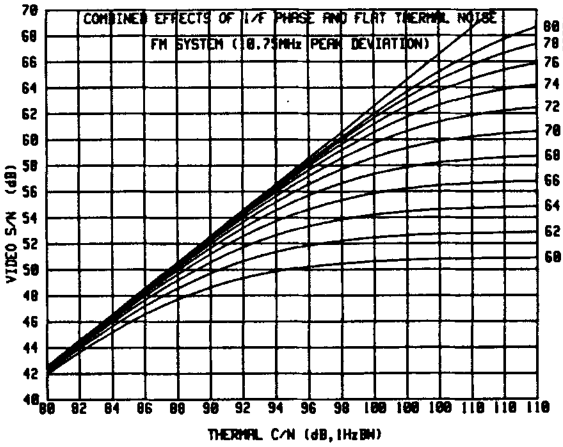

FIGURE 4

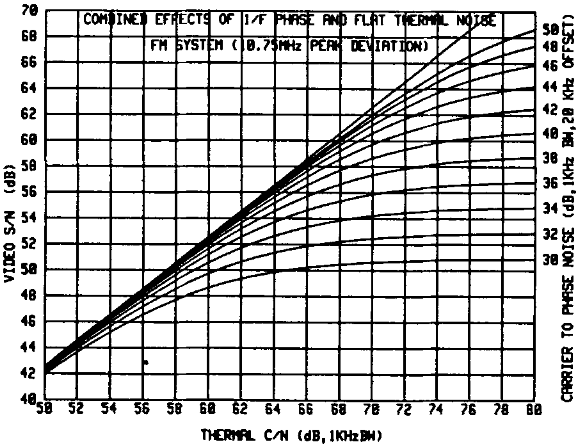

FIGURE 5

MEASUREMENT

A series of tests were run with the objective of determining the subjective effects of phase noise and whether weighted signal to noise adequately describes the effects. These tests were also compared to the above theoretical treatment. The test equipment diagram is shown in FIG 6.

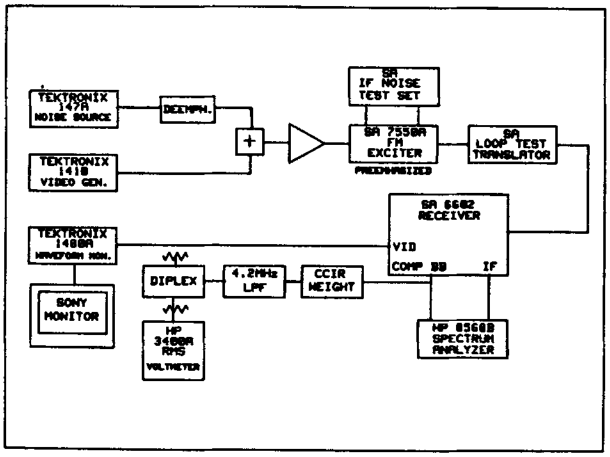

FIGURE 6 '

This system allowed addition of relatively arbitrary amounts of thermal and phase noise. The results of these tests are summarized in Table 1. Agreement with theory is quite good.

The tests for perceptibility were all run over a relatively short time span with a group of five expert observers. The tests were not intended to establish absolute perceptibility levels but to compare phase noise and thermal noise. Perceptibility was measured by switching between the lowest possible noise conditions and the test condition. Both thermal and phase noise were perceptible at the same weighted video signal to noise.

THERMAL C/N	C/N (PHASE) @ 20 KHz OFFSET	S/N MEAS	S/N CALC	DIFF
110	77.9	79.4	67.3	12.1
95	77.9	57.6	57.2	0.4
90	77.9	52.7	52.5	0.2
88	77.9	50.7	50.5	0.2
86	77.9	48.6	48.5	0.1
84	77.9	45.9	46.5	-0.6
110	69.8	59.5	60.4	-0.9
110	65.0	55.5	55.8	-0.3
110	61.3	51.7	52.2	-0.5
110	51.1	42.0	42.0	0.0
90	69.8	51.9	51.9	-0.0
90	65.0	50.9	50.9	0.0
90	61.3	49.2	49.4	-0.2
90	51.1	41.7	41.6	0.1

TABLE 1

With the noise level high enough to be clearly visible, the appearance of phase noise and thermal noise was different. This was also true when measuring noise using a waveform monitor. The reason for this is due to the difference in the baseband noise spectra as seen in Fig's 4 & 5. The combination of the flat phase noise spectrum and the deemphasis causes the low frequency phase noise to dominate. Thus the appearance of the noise in the picture is more "streaky" with phase noise. On the waveform monitor, the low frequency components cause the noisy line to "wobble" as well as the usual "fuzzy" look of thermal noise.

CONCLUSION

If accurate measurements of the phase noise of various components in an FM link are made and the thermal carrier to noise is known, a very good calculation of the video signal to noise can be made.

While the appearance of phase noise in the picture is generally different from thermal noise, the level of perceptibility occurs at the same Weighted Signal to Noise Ratio. The Weighted Signal to Noise is then a good measure of the effect of phase noise in general. When referring to the literature, conclusions drawn about the observable levels of noise will apply to phase noise as well as thermal noise.

In considering new formats, we must evaluate their differences from NTSC. In particular, where chroma is carried in a different manner and where added subcarriers are used, the effects may well be unlike that observed for NTSC. The deemphasis will also vary with format.

REFERENCES

1. Rezin Pidgeon and Dan Pike, "Oscillator Phase Noise and Its Effects in a CATV System", NCTA 37th Annual Convention, Los Angeles, 1988

2. Members of the Technical Staff, Bell Telephone System, Transmission Systems for Communications, Fourth Edition, 1970 Section 5.2

3. T. M. Straus, "Defining S/N", Communications Engineering and Design, February 1986, p22

4. Hewlett-Packard, "Understanding and Measuring Phase Noise", Application note 207, October 1976