MECHANICS OF AERIAL CATV PLANT

Tim V. Dugan

Times Fiber Communications, Inc.
358 Hall Avenue, Wallingford, CT 06492

ABSTRACT

Understanding the mechanics of aerial cable installation is essential to the cable engineer for proper plant design and for maximum cable plant reliability and longevity. A key part of that understanding is the calculation of sags and tensions. Although basic sag and tension equations are available, little is available on calculating sag and tension with changing temperature and load.

This paper extends the basic sag and tension equations to address these effects, and gives the ability to solve integrally supported (Figure 8) cable, tighly lashed cable, and unequal elevation problems. The equations are applied to several basic tension and clearance problems and further, to analyze expansion loop life, tight lashing, cable buckling, and center conductor pullouts. Obscure cable failure modes caused by wind gusting, solar heating and radiative cooling are discussed.

INTRODUCTION

The calculation of sag and tension can be quite useful to the cable television engineer. The calculations are used to select the appropriate size support strand for a given application or to determine if clearance requirements are met. They serve as an aid to the cable designers who determine the mechanical stress that the cable must be capable of withstanding or to evaluate such problems as expansion loop cracks and center conductor pullouts. The following is a discussion of sag and tension calculations which develops the simple case of a single wire suspended between two supports at equal elevation and works up through multiple elements, which takes into consideration each of the cable's components, with various loads and temperatures at unequal elevations.

BASIC EQUATIONS: WIRE, EQUAL EVALUATIONS

This first section covers the basic equations that apply. The sections that follow describe the application of these concepts. Figure 1 shows a single span of cable suspended between two fixed supports at equal elevations.

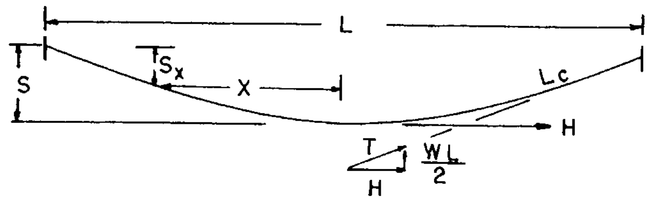

FIGURE 1.

$$H = WL^2/8S \qquad (1)$$

$$L_c = L + 8S^2/3L \qquad (2)$$

Where:

H = Horizontal component of tension (lb)
W = Linear weight of the wire (lb/ft)
L = Distance between supports (ft)
S = Sag (ft)
L_c = Actual length of wire (ft)

These equations[1] are a parabolic approximation of the actual form that a flexible wire assumes, which is a catenary. The equation for a catenary is:

$$S = (H/W) \ [\cosh (WL/2H)-1] \qquad (3)$$

$$\approx WL^2/8H$$

The difference between the catenary and the parabola is that the parabola is approximately ½% smaller if the sag is about 6% of the span[1]. Generally, the sag is less than 2% for most cable television applications.

Another useful equation can be used to find the sag at any point along the span:

$$S_x = S(1-4x^2/L^2) \qquad (4)$$

UNEQUAL ELEVATIONS

The wire will assume the catenary form (or the parabolic approximation of this form) regardless of where the supports are located. The supports simply apply an equal and opposite vertical and horizontal force on the cable. Thus, the supports can be anywhere on the curve. Of course, after the load or temperature or both change, the sag changes. The supports are still on a parabola, but the parabola is different.

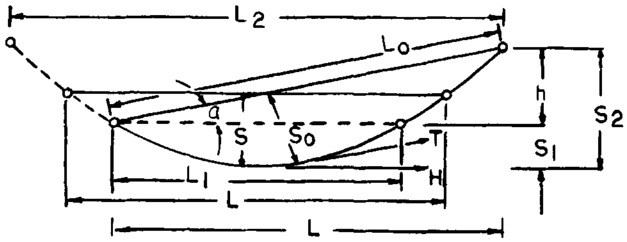

FIGURE 2

$$S = S_o = WL^2 \cos\alpha/8T$$

$$= WL^2/8T\cos\alpha = WL^2/8H \qquad (5)$$

$$L_1 = L-2hH/WL \qquad (6)$$

$$L_2 = L+2hH/WL \qquad (7)$$

$$S_1 = WL_1^{\,2}/8H \qquad (8)$$

$$S_2 = WL_2^{\,2}/8H \qquad (9)$$

$$L_c = L+(4/3)[(S_1^{\,2}/L_1) + (S_2^{\,2}/L_2)] \qquad (10)$$

Where:

T = Tension in the direction of the cable (lb)

With these equations[1] (with Eqs. 6 and 7 slightly modified from the referenced source) the initial sag or tension can be calculated. Of course, one or the other must be known and span length and cable weight must also be given. They can be used, for instance, to calculate clearance over roads and sidewalks, and to determine specific size strand to be used under worst case loading. Note, however, that there is no mention of the properties of the materials such as their thermal coefficient of linear expansion or their elastic modulus. Indeed, these equations are correct but only for the initial conditions.

TEMPERATURE AND LOAD CHANGES

There are other conditions which encompass a range of temperatures and loads. For example, what if the amount of installed sag is fixed at some maximum value, and then the span is subjected to a different temperature and a load which may exceed the strand's strength? Or, if the sag and tension is known for the worst case load, what would the stringing sag and tension be at a different temperature and load? To solve these types of problems, further knowledge of the materials that are used is needed and the above equations must be adapted.

Suppose the wire is installed at some initial temperature (T_o) and the cable has a linear weight (W_o). Then the temperature drops to some final temperature (T_f) and the linear weight of the cable changes to some final weight (W_f) because, perhaps, ice has accumulated and a strong wind is blowing. What happens to the sag and tension?

As the temperature drops, the cable tries to get shorter because of its expansion coefficient, but because the span length is fixed the sag must get smaller, thus increasing the tension. The tension also affects the length of the cable due to its elastic modulus. So, as the tension increases, the length of the cable increases thus minimizing the effect. On one hand the cable tries to get shorter because of temperature, but it cannot get as short as it would like because the tension is increasing. Finally, the increased load causes the tension to increase, but because the cable is elastic it elongates in its elastic region thus increasing the sag and reducing the tension. All of this occurs simultaneously, and a balance or equilibrium is continually being maintained.

To determine the final sag and tension a mathematical description of these events will be given.

Unstressed Length

The most important quantity in the process of determining the final sag and tension is the cable's initial unstressed length at the initial temperature. The cable's unstressed length is the length of the cable if it had no tension or stress on it. It should not be confused with L_c, the actual cable length.

The unstressed cable length is necessary to know because, as additional tension is applied to the cable, its stressed length changes as a function of its unstressed length (and also its elastic modulus). To determine the appropriate amount of expansion and contraction due to temperature changes, again the unstressed length must be used (along with its thermal coefficient of linear expansion).

The stressed length of the cable (L_c) can be determined from Eq. 2 or 10. From this the unstressed length can be determined. Assuming that the material is elastic and follows Hooke's Law, its strain is proportional to the stress applied by a factor called the elastic modulus.

$$E = \sigma/e \qquad (11)$$

$$= FL/A\,\Delta L \qquad (12)$$

Where:
E = Modulus of elasticity (psi)
e = Strain = $\Delta L/L$ (dimensionless)
σ = Stress = F/A (psi)
ΔL = Change in length as the result stress (ft)
F = Force (lb)
A = Cross sectional area perpendicular to the force applied (sq in)

$$L_c = L+\Delta L \qquad (13)$$

$$= L+FL/AE \qquad (14)$$

$$L_c = L(1+F/AE) \quad (15)$$

It is assumed that the tension is the same along the entire length, but, because the wire has a finite weight per unit length, the ends of the wire near the support have more tension. The horizontal component of the tension is the same everywhere in the wire. The vertical component of the tension is zero at the lowest point since there is zero cable weight. The vertical component of tension at the support is the weight of the cable between the support and the lowest point.

The unstressed length, then is:

$$L_{uo} = L_c/[1+(H/AE)] \quad (16)$$

It should be noted that this only applies for stresses within the elastic limit of the material. Also, the elastic modulus of the material is not necessarily constant with temperature; therefore, a knowledge of the characteristics of the materials is important.

TEMPERATURE CHANGES

Once the unstressed length has been determined at the initial temperature, the unstressed length can be determined at other temperatures from the following:

$$L_{uf} = L_{uo}\,[1+a(T_f-T_o)] \quad (17)$$

Where:
L_{uf} = Final unstressed length at T_f (ft)
L_{uo} = Original unstressed length at T_o (ft)
a = Thermal coefficient of linear expansion (in/in F)
T_f = Final temperature (F)
T_o = Original temperature (F)

It should be noted that the thermal coefficient is not necessarily constant and again it is necessary to be familiar with the properties of the materials.

Most materials have positive thermal coefficients of linear expansion which cause them to get longer when the temperature increases. When the temperature decreases, they get shorter. (In some cases, materials are specially selected because they have negative expansion coefficients so that when used in conjunction with other materials the overall change in length is minimized or matched to some other component such as in fiber optic transmission lines.)

There are several factors that affect temperature. The temperature of the cable or wire is of course affected by the air temperature. The air temperature changes on a daily basis (diurnal) as well as yearly basis (annual). If electrical current is carried through the wire, a temperature rise will also occur, but, generally speaking, this rise is small for CATV cable. Another factor is radiation. During the day the cable is heated from the sun. At night the cable radiates its heat toward clear skies. The wind tends to minimize this affect.

Measurements made in Connecticut on the longest day of the year when the sun's rays are most direct indicate that a temperature rise of about 45F above ambient can be expected on black jacketed cable and about 24F rise above ambient for unjacketed aluminum sheathed coaxial cable. At night, in the same area, the temperature of black jacketed cable was about 8F below ambient and 4F below ambient for unjacketed aluminum cable.

LOAD CALCULATIONS

Before the final sag and tension can be calculated, it is necessary to evaluate the span of wire in terms of load. Initially we assumed that the wire's weight was the only load applied. Under worst case conditions, the wire may have ice formed around its circumference and at the same time a strong wind blowing on it. The total load on the wire is the resultant of all the vertical and horizontal loads. (The horizontal load here is perpendicular to both the vertical and the wire itself.)

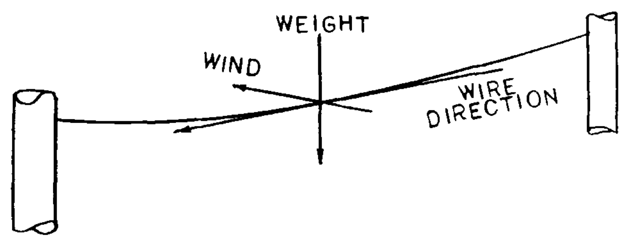

FIGURE 3

The total resultant load of the wire is the vector sum of all the horizontal and vertical components:

$$W_f = [(\Sigma W_V)^2 + (\Sigma W_H)^2]^{1/2} \quad (18)$$

Where:
W_f = Final linear cable weight (lb/ft)
ΣW_V = Sum of all vertical weights (lb/ft)
ΣW_H = Sum of all horizontal forces (lb/ft)

As an example, the vertical weight components may include: the weight of the support strand, the weight of all the cables, the weight of the lashing wire and possibly a cylinder of ice around the group. The horizontal force is only attributed to wind loading. In some cases a weight constant (e.g. in heavy loading districts, 0.3 lb/ft) is added to the resultant final weight for safety.

Ice Loading

The additional weight caused by ice build-up on the cable is usually calculated based on a hollow cylinder having an inside diameter equal to the outside diameter of the bundle of the cables and the support strand and with a given thickness which is usually 0.25" or 0.5" depending on which loading district is used. Using 57 lb/cu ft as the weight of ice[3], the linear ice weight around the cable can be calculated from:

$$W_{ice} = 1.244(D_i t+t^2) \tag{19}$$

Where:
W_{ice} = Linear ice weight (lb/ft)
D_i = Diameter over cable bundle (in)
t = Thickness of the ice (in)

Wind Loading

The wind loading on a circular surface can be calculated from[3]:

$$P = 0.00256V^2 \tag{20}$$

Where:
P = Horizontal wind pressure (lb/sq ft)
V = Wind velocity (mph)

In order to use this equation, the pressure must be converted to a load or force on the projected surface area of the bundle that faces the wind. Assuming that the wind is perpendicular to the cable and we wish the results to be in terms of lb/ft:

$$F_{wind} = PD_o/12 \tag{21}$$

Where:
F_{wind} = Wind loading (lb/ft)
P = Wind pressure (lb/sq ft)
D_o = Diameter of cable bundle (in.) (including ice if appropriate)

It should also be pointed out that horizontal loading results in sags which are not vertically directed. The horizontal and vertical components of the final sag can be resolved since they are proportional to the horizontal and vertical loads described in Eq. 18. i.e.

$$S_f = (S_V^2 + S_H^2)^{1/2} \tag{22}$$

So, for example the actual vertical component of the sag would be:

$$S_v = S_f(\Sigma W_v)^2/[(\Sigma W_v)^2+(\Sigma W_H)^2] \tag{23}$$

FINAL SAG AND TENSION

From Eq. 17 the final unstressed length of the wire can be determined based on the final temperature. From Eq. 15 we can determine the actual length of the wire if its tension is known:

$$L_{cf} = L_{uf}\ (1+H_f/AE) \tag{24}$$

Where:
L_{cf} = Final stressed length of the wire (ft)
L_{uf} = Final unstressed length of the wire (ft)
H_f = Final horizontal tension (lb)
A = Cross sectional area of the wire (sq in)
E = Elastic modulus, tensile (psi)

If the supports are at equal elevations, the final length of the wire is also (from Eq. 2):

$$L_{cf} = L+8S_f^2/3L \tag{25}$$

Where:
S_f = Final sag (ft)

This is actually a special case of supports at unequal elevations. The following progression puts Eq. 10 into the same form as Eq. 25. Consider:

$$L_c = L+(4/3)[S_1^2/L_1) + (S_2^2/L_2)] \tag{26}$$

From Eqs. 5, 6, and 7, S_1 and S_2 can be set in terms of S as:

$$S_1 = SL_1^2/L^2 \tag{27}$$

$$S_2 = SL_2^2/L^2 \tag{28}$$

So Eq. 26 can be transformed to:

$$L_c = L+(4S^2/3L^4)(L_1^3+L_2^3) \tag{29}$$

From Eqs. 8 and 9, L_1 and L_2 can be stated in terms of S and h and L as:

$$L_1 = L(1-h/4S) \tag{30}$$
$$L_2 = L(1+h/4S) \tag{31}$$

After a bit of work we find:

$$L_1^3+L_2^3 = L^3(2+3h^2/8S^2), \tag{32}$$

which can be plugged into Eq. 29 to yield:

$$L_{cf} = L+(8S_f^2/3L) + (h^2/2L). \tag{33}$$

Notice when h=0, Eq. 33 reduces to Eq.25, the equal elevation equation.

Back to the problem at hand, we were in search of another equation to set equal to Eq. 24; our search is over with Eq. 33. Actually Eq. 24 would be better in terms of S_f, like Eq. 33. This is easily done via Eq. 1.

$$L_{cf} = L_{uf}[1+(W_f L^2/8S_f AE)] \tag{34}$$

Setting Eq. 33 and 34 equal to each other yields:

$$L+(8S_f^2/3L) + (h^2/2L) - L_{uf} - (L_{uf}W_f L^2/8S_f AE) = 0 \tag{35}$$

Which can be put into the form of:

$$S_f^3+S_f(3L/8)(L+h^2/2L-L_{uf}) - (3L/8)(L_{uf}W_f L^2/8AE) = 0 \tag{36}$$

Notice that every variable in Eq. 36 is defined except the final sag (S_f). This equation takes into account a new temperature and a new load. S_f can be found via the solution of the cubic equation. Eq. 36 can be rewritten as:

$$S_f^3 + aS_f + b = 0 \quad (37)$$

Where:

$$a = 3[L^2 + (h^2/2L) - LL_{uf}]/8 \quad (38)$$

$$b = -3W_f L^3 L_{uf}/64AE \quad (39)$$

The solution of this particular form of cubic equation is as follows[4]:

$$\text{If} \quad (a/3)^3 + (-b/2)^2 \geq 0 \quad (40)$$

Then

$$S_f = \left\{(-b/2) + [(a/3)^3 + (-b/2)^2]^{1/2}\right\}^{1/3} + \left\{(-b/2) - [(a/3)^3 + (-b/2)^2]^{1/2}\right\}^{1/3} \quad (41)$$

Note, if the above condition is met and since -b/2 is always positive, the result will always be a real root.

$$\text{If} \quad (a/3)^3 + (-b/2)^2 < 0 \quad (42)$$

Then

$$S_f = 2(a/3)^{1/2}\cos\left\{(1/3)\cos^{-1}[(-b/2)/(a/3)^{3/2}]\right\} \quad (43)$$

Note, if Eq. 42 is true and since (-b/2) is always positive then (a/3) is certainly negative and again a real root will be obtained.

Once the final sag is found the final tension is easily found from:

$$H_f = W_f L^2/8S_f \quad (44)$$

CLEARANCE CALCULATIONS

Sometimes when checking to assure that the proper clearances are met (e.g. during make ready), it is necessary to know the elevation of the wire at points other than at the lowest point. For equal elevations Eq. 4 can be used. The sag at any point is given with respect to the lowest point which, for problems of equal elevation, is always exactly half-way between the supports.

For problems of unequal elevation, it is not particularly helpful to know what the sag is with respect to the position of the lowest point sag because not only is the lowest point sag not exactly half-way between the two supports, it may not be between the two supports at all!

Equation 4, however, can be adapted.

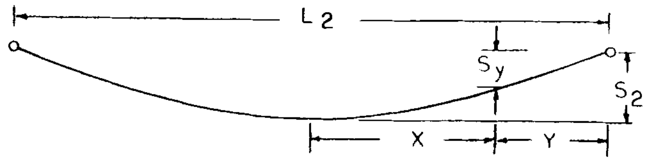

FIGURE 4

From the above figure (which is simply Fig. 2 without some of the details, but includes some new information about the sag at any point) it can be seen that x can be stated in terms of y, which is the horizontal distance from the highest support as:

$$x = (L_2/2) - y \quad (45)$$

Where:
- x = Distance from the lowest point (ft)
- y = Distance from the highest point (ft)
- L_2 = Span length (ft)
- S_2 = Sag at lowest point (ft)

Substituting Eq. 45 into Eq. 4 we obtain:

$$S_y = S_2 \left\{1 - 4[(L_2/2) - y]^2/L_2^2\right\} \quad (46)$$

Where:
- S_y = Sag at y from the highest support (ft)

Equation 46 can also be used for equal elevation problems. Notice from Eq. 9 that L_2 is not fixed. L_2 will be different from its original value depending upon the tension (H). Therefore, it is necessary to determine both the final L_2 and S_2 if clearance is to be calculated.

Creep

It should be noted that materials, when exposed to stress even if below their elastic limit for a long period of time, tend to deform and take a permanent set. This slow acting deformation is called creep. For clearance calculations that require exacting accuracy, factors such as creep must be taken into consideration and creep data on the material used to support the cable should be obtained. Although it is beyond the scope of this paper to provide such information, two data points are given for discussion. The creep rate on extra high strength galvanized steel wire and strand is about 0.08% at 70% of the wire's rated breaking load. This much creep can occur in 24 hours, but very little additional creep takes place over the next 1000 hours of exposure, perhaps a total of 0.09%.

To illustrate the implications of this "small" number, consider a 100 ft span with 1 ft of sag. Suppose during the course of time the strand is exposed to such stress that would cause 0.09% creep. The implication can be calculated from Eq. 2. The resulting sag would be 2.07 ft instead of returning to 1 ft as might be expected. Although the strand should not be exposed to such stresses, the purpose of this illustration is to show the limitations of the calculations and highlight the need for caution and good engineering judgement.

SPECIAL CONFIGURATIONS

Multiple Elements

In some cases the wire may be comprised of several different materials with different cross sections. Cable with integral messengers ("Figure 8") drop cable are some examples. Each material has its own expansion coefficient and elastic modulus. When suspended, each element takes a different portion of the total load. The amount of the load that any particular cable element takes is a function of its elastic modulus and area as compared to the total resultant modulus of all the materials and the total area of all the materials.

The resultant modulus can be found by multiplying the modulus of each material by the ratio of the material's area to the total area:

$$E_r = (A_1/A_T)E_1 + (A_2/A_T)E_2 + (A_3/A_T)E_3 + \ldots + (A_n/A_T)E_n \qquad (47)$$

Where:
E_r = Resultant elastic modulus (psi)
A_T = Total cross sectional area of the wire (sq in)
A_1 = Cross sectional area of element 1 (sq in)
A_2 = Cross sectional area of element 2 (sq in)
A_3 = Cross sectional area of element 3 (sq in)
A_n = Cross sectional area of element n (sq in)
E_1 = Elastic modulus of element 1 (psi)
E_2 = Elastic modulus of element 2 (psi)
E_3 = Elastic modulus of element 3 (psi)
E_n = Elastic modulus of element n (psi)

These values (i.e., E_r and A_T) along with the following value for expansion coefficient can be directly substituted into the above equations for sag and tension wherever E, A, and a occur.

The resultant coefficient of linear expansion (a_r) is calculated as follows:

$$a_r = (A_1E_1a_1 + A_2E_2a_2 + \ldots + A_nE_na_n)/(A_1E_1 + A_2E_2 + \ldots + A_nE_n) \qquad (48)$$

By using these values for E_r, A_r, and a_r the final sag and final tension can be determined at the new temperature with the new load.

Because the above equations which include elastic modulus only work when the materials are stressed within their elastic region, it is important to make sure that each element's elastic limit is not exceeded.

The unstressed length of all the elements is the same only at the original temperature. It is assumed that any stress built into the cable is small compared to the installed stress. Once the temperature changes, each element expands and contracts at its own rate. Also, each element has its own elastic modulus and cross sectional area. Unless the elements are bonded together in some fashion they may move independently of one another.

In order to determine the stress on any particular element, the unstressed final length must first be determined. It can be calculated, based on the change in temperature and the expansion coefficient of the element in question, as follows:

$$L_{ufn} = L_{uon}[1+a(T_f-T_o)] \qquad (49)$$

Where:
L_{ufn} = Final unstressed length of any particular element n (ft)
L_{uon} = Original unstressed length of any particular element n (ft)

Note: L_{uon} is the same for all elements and exactly the same as that given in Eq. 16.

The final cable length (under stress) is the same for all elements. It can be determined from either Eq. 2 or Eq. 10. The strain that the cable element is under is then:

$$e_n = (L_{cf} - L_{ufn})/L_{ufn} \qquad (50)$$

the stress is the modulus times the strain:

$$\sigma_n = E_n(L_{cf}-L_{ufn})/L_{ufn} \qquad (51)$$

And the tension on element n is:

$$H_n = A_nE_n(L_{cf}-L_{ufn})/L_{ufn} \qquad (52)$$

Where:
H_n = Tension on element n (lb)
A_n = Cross sectional area of element n (sq in)
E_n = Elastic modulus of element n (psi)
L_{cf} = Final cable length (ft)
L_{ufn} = Unstress final length of element n (ft)

The stress in Eq. 51 should be tested to assure that the elastic limit not be exceeded. Aside from exceeding the elastic limit of any particular element, one additional caution should be noted. Generally, "Figure 8" cable used in CTV applications is for distribution purposes and so taps are installed. To install a tap the cable must be cut and separated from the messenger wire. Cutting the cable relieves the stress in the cable at the point where it is cut. This tension, however, does not necessarily disappear. A significant portion, if not all of it, is diverted to the steel messenger wire. So, wherever taps are installed in messenger cable, you must assure that the total tension does not exceed the safe limits of the messenger if it were alone.

Cable Tightly Lashed

In cases where cable is lashed to the strand so tight as to restrict cable from moving independently of the steel strand support, a similar analysis can be performed. This condition can occur even if the lashing is not restrictive: by having no expansion loops.

We will call this third case "cable restrained." In this case, the steel strand is installed and loaded with the cable. The cable is then lashed so tight that cable movement is restricted. The resultant condition is that the steel strand is under stress and the cable has zero stress.

After the temperature, load or both change, the load is not distributed like it was in the case of "Figure 8" cable. They differ in that there is no original cable tension; so the original unstressed cable length is the same as the actual cable length (L_{co}). The original unstressed steel length is slightly shorter.

It will be assumed that the cable and steel support strand are bound together, either by tight lashing or lack of expansion loops. This may not be the case because some slight differential movement between the cable and support strand can be expected, but it will allow us to evaluate the extreme conditions; the actual[5] lying somewhere between this and the first case (i.e., the steel taking the full load at all temperatures and under loading conditions).

The first step in solving this problem is to find the unstressed length of the composite assembly. It is certainly not the unstressed strand length because the two components, the steel and the cable, are bound together. Nor is it the unstressed cable length. The unstressed length of the composite is somewhere between the two.

When the composite assembly is unstressed, an interesting condition occurs; the steel is under tension and the cable under compression.

Again, as before, the actual length of the assembly initially is found from Eq. 2.

$$L_c = L+8S^2/3L \tag{52}$$

The unstressed length can be calculated from:

$$L_u = L_c/[1+(H/A_T E_r)] \tag{53}$$

E_r is found in the same way as for Figure 8 cable, Eq. 47. The tension, H, in Eq. 53 is the tension on the steel support alone. The original unstressed length can be used to find the final unstressed length at the final temperature from Eq. 17 except that the resultant expansion coefficient (a_r) should be used in place of a. Finally, the final sag can be calculated from Eq. 41 or 43. The total tension on the assembly can then be found from Eq. 44.

The original tension on the components was easily determined; the steel had all the tension, the cable had none. At final state, that is, after the load, temperature or both change, the tension on each component may not be quite so easy to determine. We will divide the tensions into two groups: namely the tension on the steel and the tension on the other components.

To analyze the final tension on the steel, we must first determine its initial unstressed length. It is:

$$L_{uo\ steel} = L_{co}/[1+(H_o/A_{steel}E_{steel})] \tag{54}$$

Changing the temperature altered this length to:

$$L_{uf\ steel} = L_{uo\ steel}[1+a_{steel}(T_f-T_o)] \tag{55}$$

The stress on the steel is then:

$$\sigma_{steel} = E_{steel}(L_{cf}-L_{uf\ steel})/L_{uf\ steel} \tag{56}$$

The tension on the steel is:

$$H_{f\ steel} = A_{steel}\sigma_{steel} \tag{57}$$

The stress and tension on the other elements can be found in a similar manner except that the original unstressed cable length equals the original actual cable length.

$$L_{co} = L_{cuo} \tag{58}$$

The change in length of any particular element other than the steel can be determined from:

$$L_{ufn} = L_{co}[1+a_n(T_f-T_o)] \tag{59}$$

Where the subscript "n" refers to one of the cable elements.

The stress on that particular element is then:

$$\sigma_n = E_n(L_{cf}-L_{ufn})/L_{ufn} \tag{60}$$

And the tension on that element is:

$$H_{fn} = A_n\sigma_n \tag{61}$$

Unfortunately, it is difficult to first assume full load at the lowest temperature for restrained cable. For example, you may want the tension on the strand to be 60% of its breaking strength at full load at the lowest temperature. The tension on the other components can be anywhere between zero and the material's elastic limit, depending on what the original temperature was.

In the other two cases, the original temperature and load have no restrictions. But in this case, cable restrained, the original conditions must be at the temperature at which the cable was lashed to the strand. Of course, if enough information is known about the final conditions, the initial conditions can be determined.

In review, three basic configurations have been discussed with regard to sag and tension. The first was a simple case of a single wire. The second considered multiple elements which can be applied to "Figure 8" type cable or drop wire. The last, "cable restrained", addressed what happens when the cable has zero tension initially but supports some of the load as conditions change. Each configuration was considered after the temperature, load or both changed.

APPLICATIONS

Strand Tension Under Worst Case Load

To illustrate the application of these equations a few examples will be given. Consider, first, a 125 foot span having a 500 and 750 size jacketed cables supported by a 1/4" EHS strand. Assume that the cable has expansion loops and is free to move independently of the steel support strand so that there is no tension on the cable under any condition. The cable was installed at 60F and has a 1.5% sag (1.875 ft). The cable is installed in a heavy loading district with extreme wind loading of 21 lb/sq ft. The question is, what is the tension on the strand under worst case conditions?

From Table 1 and 3, the total weight of the strand, plus the 500J, plus the 750J, plus the double lashing is 0.439 lb/ft. The maximum tension on the strand must not exceed 60% of its rated break strength under worse case expected loading. The initial strand tension and cable length at 60F can be calculated from Eq. 1 and 2.

$$H = (0.439\ \text{lb/ft})\ (125\ \text{ft})^2/8(1.875\ \text{ft}) = 457.3\ \text{lb}$$

$$L_c = 125\ \text{ft} + 8(1.875\ \text{ft})^2/3(125\ \text{ft}) = 125.0750\ \text{ft}$$

The unstressed length of the steel support strand is found from Eq. 16 and the elastic modulus of the strand from Table 2 and 3.

$$L_u = (125.0750)/[1+457.3/(0.035185)\ (28X10^6)] = 125.01697\text{ft}$$

Now the unstressed length at any other temperature can be determined. From Table 4, the temperature for a heavy loading district is 0F. From Eq. 17 and the expansion coefficient of steel in Table 2, the unstressed length of the steel at 0F is:

$$L_{uf} = (125.01697)[1+7.2x10^{-6}(0-60)] = 124.96296\ \text{ft}$$

Assuming that the cables are stacked one on top of the other and from the dimensions in Table 1 and 3, the width of the cable plus the strand is 1.620 in. From Eq. 19, 0.5 in thick ice over the cable weighs 1.319 lb/ft. So the total vertical weight (i.e., ice plus the cable) is 1.757 lb/ft. See Fig. 5.

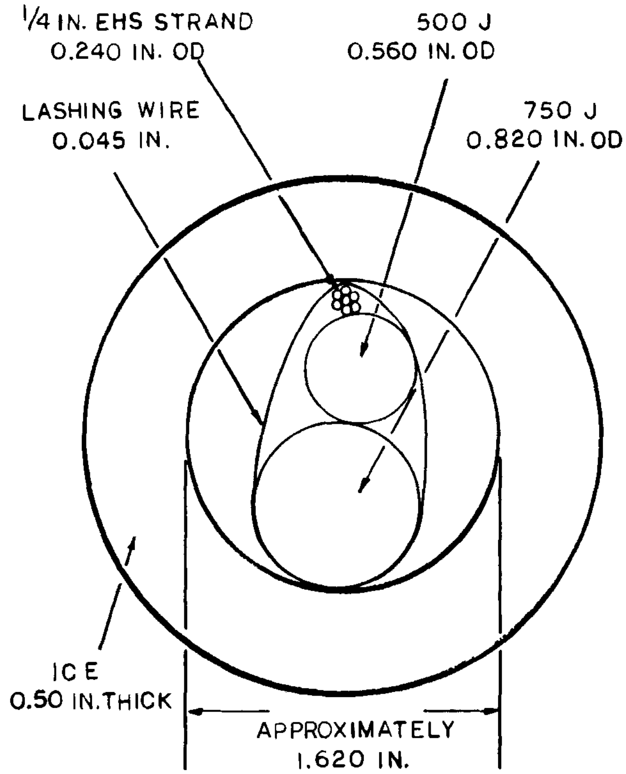

FIGURE 5

The horizontal load from the wind is calculated from Eq. 22. The projected diameter over the ice is 2.620 in (i.e. 2x0.5in + 1.620in). The wind pressure for a heavy loading district, from Table 4, is 4 lb/sq ft. The loading is then:

$$F_{wind} = (4)\ (2.620)/12 = 0.873\ \text{lb/ft}$$

The vector sum of the horizontal and vertical loads calculated from Eq. 18 is:

$$W = [(1.757)^2 + (0.873)^2]^{1/2} = 1.963\ \text{lb/ft}$$

The weight adder for a heavy loading district is 0.30 lb/ft and results in a final weight of:

$$W_f = 1.963 + 0.30 = 2.263\ \text{lb/ft}$$

To solve for the final sag in Eq. 37, a and b are first found from Eq. 38 and 39. The difference in support elevations, h, is zero so:

$$a = 3[(125)^2-(125)\ (124.96296)]/8 = 1.73608875$$

$$b = -3(2.263)(125)^3(124.96296)/64(0.035185)(28x10^6) = -26.27575$$

$$(a/3)^3 = 0.19380$$

$$(-b/2)^2 = 172.604$$

Clearly, Eq. 40 is true and the final sag (which is not vertically directed) can be found from Eq. 41 and the final tension from Eq. 1.

$$S_f = 2.779 \text{ ft}$$

$$H = (2.263)125)^2/8(2.779)$$
$$= 1590 \text{ lb}$$

This is the tension (horizontally directed) in the strand under heavy loading conditions. This is well within the 3990 lb maximum limit.

One more check is necessary and that is under extreme wind loading at 60F. From Figure A2 the wind loading is 21 lb/sq ft. The resulting wind load from Eq. 22 is:

$$F_{wind} = (21)\ (1.620)/12$$
$$= 2.835 \text{ lb/ft}$$

By adding the horizontal and vertical load components, the final load is 2.869 lb/ft. Using the unstressed length at 60F:

$$(a/3)^3 = -0.0186452$$

$$(-b/2)^2 = 277.746$$

The final sag and tension is found in the same way as before.

$$S_f = 3.30 \text{ ft}$$
$$H = 1698 \text{ lb}$$

So the strand tension under extreme wind loading is well under the 3990 lb limit.

The above example was carried out in several steps:

1. Calculate the initial tension from Eq. 1.
2. Calculate the initial cable length from Eq. 2.
3. Calculate the initial unstressed cable length from Eq. 16.
4. Calculate the final unstressed cable length from Eq. 17.
5. Calculate the final load from the loading tables and Eqs. 18 and 19.
6. Calculate the final sag from either Eq. 41 or 43.
7. Calculate the final tension from Eq. 44.

For unequal elevations problems, L_c is calculated from Eq. 10 and h is used in Eq. 38. These equations can be loaded into a computer or programmable calculator for easy calculations.

Clearance Over A Traffic Light

Consider, next, a situation where you wish to keep a 30" clearance, under all expected conditions, over a traffic light. If the steel support is installed so that it is loaded to about 5% of its rated strength will the proper clearances be met? Figure 6 shows a sketch of the span.

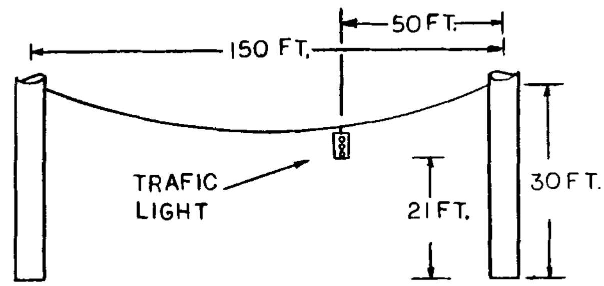

FIGURE 6

The following information is available.

SUPPORT STRAND:

1/4" EHS

$E = 28\text{x}10^6$ psi

$A = 0.035185$ sq in

$a = 7.2\text{x}10^{-6}$ in/in F

CABLE:

1 - 750J

1 - 500J

Double Lashed

Diameter = 1.620 in

Weight = 0.439 lb/ft

The cable is installed in a heavy loading district with 16 lb/sq ft extreme wind.

The initial (loaded) conditions from Eq. 1, 2, 16 and 46 are:

$H = 330$ lb $\quad L_u = 150.198554$ ft

$S = 3.74$ ft $\quad S_y = 3.33$ ft at 50 ft from the support.

Assuming that the worst case is at 32F with 0.5 in ice, no wind and a 0.3 lb/ft weight constant, the following results are obtained:

$W_f = 2.0576$ lb $\quad L_{uf} = 150.168274$ ft

$H_f = 1280$ lb $\quad S_y = 4.02$ ft

$S_f = 4.52$ ft

Assuming Heavy Load Conditions:

$W_f = 2.263$

$H_f = 1431$

$S_f = 4.45$

Assuming 120F with no load:

$H_f = 0.439$ lb/ft

$H_f = 296$ lb

$S_f = 4.17$ ft

For heavy loading 1431/1b is about 22% of the strands rated strength of 6650 lb. The expected creep is approximately 0.015%. Doubling it for margin, 0.03%,

$L_{cf} = 150.363381$ ft

This is a first order approximation of the increased sag over time due to creep. The sag in the middle of the span would be about 4.783 ft instead of 4.52 ft. As long as the attachments are: 21 + 2.5 + 5 = 28.5 ft high, there should not be any clearance problems.

Tension on Composite Materials

Consider a 150 ft span at 60F with 0.5% sag (0.75 ft). In this example let us assume that a single 750 non-jacketed cable is either tightly lashed to the strand or has no expansion loops so that in can not move independently of the 1/4" EHS support strand.

L = 150 ft
S = 0.75 ft
W = 0.299 lb/ft

COMPONENT	MAT	A (sq in)	E (psi)	a (in/in F)
STRAND	STEEL	0.03585	28×10^6	7.2×10^{-6}
SHIELD	AL	0.08075	10×10^6	12.7×10^{-6}
DIEL.	T4+FOAM	0.33965	42×10^3	48×10^{-6}
CENTER	CuCladAl	0.02138	10.6×10^6	12.2×10^{-6}
RESULTANT		0.476965	4.26×10^6	10.4×10^{-6}

The resultant is calculated from Eqs. 47 & 48. The condition at 60F are:

$S = 0.75$ ft $\quad H_{Steel} = 1.121$
$H_{Total} = 1.121$ lb $\quad H_{Alum} = 0$ lb

Assume the temperature drops to - 40F.

$S_f = 0.27$
$H_{Total} = 3{,}129$ lb

	TENSION	Eq. #	STRESS
H_{Steel} =	1,775 lb	(Eq. 57)	50,448 psi
H_{Shield} =	980 lb	(Eq. 61)	12,136 psi
H_{Diel} =	111 lb	(Eq. 61)	327 psi
H_{Center} =	264 lb	(Eq. 61)	12,348 psi

The stresses on the shield and center conductor have exceeded their elastic limits. This translates to excessive stress on the conductors. This span will be prone to center conductor pullout problems and stress fatigue.

Now assume instead that the cable was installed on a cold day, say at 40F, and during the summer the cable temperature rose to 120F. The intitial sag and tension are the same as before; the final conditions are:

$S_f = 2.14$ ft $\quad H_{Steel} = 1023$ lb
$H_{Total} = 393$ lb $\quad H_{Shield} = -435$ lb
$H_{Diel} = -82$ lb
$H_{Cen} = -113$ lb

Notice that the cable is under compressive forces.

The force (F) required to cause the aluminum to buckle under compressive forces is:

$$F = \pi^2 EI/X^2$$

Where:
I = Moment of inertia
X = Unsupported distance

The moment of inertia, I, for 750 cable is approximately 0.005 in^4 so the maximum unsupported distance that the cable can be exposed to without buckling is about 34in. Although the lashing wire spacing is much less than 34 in, it is quite conceivable that, at the pole, the cable will be unsupported for over 34 in and cable buckling is imminent.

Long Spans-Cable Movement and Expansion Loops

Consider a long span, approximately 200 ft of 750 cable, with 1.5% sag at 60F. Suppose that the cable has expansion loops and the cable can move freely as the temperature changes. Consider then that the temperature changes from 50F at night to 110F during the day. What is the differential cable movement? This much temperature change is not unreasonable to assume since solar heating and radiative cooling can account for a 28F temperature change on unjacketed cable even if the ambient temperature does not change.

60F	50F	110F
L = 200		
S = 2 ft	1.89 ft	2.60 ft
H = 748 lb	791 lb	575 lb
L_c = 200.05333ft	200.04768ft	200.09026ft
L_{Cable}=200.05333ft	200.02633ft	200.18832ft
	- 0.02135ft	+0.09806ft
	- 0.256 in	+1.177 in

1.43 in TOTAL

Expansion Loop Life

The life of an expansion loop can vary significantly depending on a number of factors: the surface finish of the aluminum, the type of loop, the depth of the loop, the size of the cable, the wall thickness of the aluminum outer conductor, whether it's jacketed or not, and the excursion distance per cycle. Without going into expansion loops to any great extent, the following expansion loop lifes are typical of semiflexible cable with measured 6 in depth and total 1 in excursion (i.e., + 0.5in and -0.5 in from the neutral point). The expansion loops in 0.500 in cables were formed with a LEMCO G120 and the 1.000 in with a G240.

CYCLES TO OUTER CONDUCTOR
FRACTURE (1" EXCURSION)

TYPE	UNJACKETED	JACKETED
0.500 in	29.9K	51.8K
0.750 in	18.0K	22.5K
1.000 in	8.0K	17.9K

The life of other cable sizes can be roughly approximated by interpolation. If each cycle is equivelent to one day, then the life of 0.750 in unjacketed cable would be about 49 years. Measurements show that as you double the excursion distance, the life of the loop drops by a factor of 10 for jacketed cables and by a factor of about 20 for unjacketed cables. The depth of the loop is extremely important. For a 0.500 in unjacketed cable with a 3 1/2in depth instead of 6in, the aluminum fractures at 1.5K cycles instead of 29.9K cycles.

For the specific case given above where the loop is exposed to 1.4 in excursions, the life of the 750 cable loop is about 8 years. Therefore, even with modest temperature changes, a single expansion loop may not be adaquate for long spans.

Wind Gusting

Aside from length changes due to temperature changes, changes can also occur due to load changes. Although load changes due to ice build-up are infrequent, perhaps a few times a year, load changes due to wind can occur quite frequently especially if gusting is considered.

The frequency and amplitude distribution of wind gusting is complicated and can vary significantly from one region to the next. In general, gusting is far more severe in urban areas with buildings than in flat, level country. Although there is some good data available on wind gusting[6], the next example simplifies the frequency and amplitude distribution of gusting to a single data set.

Consider a 150 ft span of cable, one 500J and one 750J cable, suspended between supports with 1.5% sag at 60 F in a town with buildings. Consider next that the distribution of wind gusts for a one year period can be represented by 1000 gusts from 20 mph to 60 mph. The following table can be generated:

Wind Speed (mph)	Wind Force (lb/sq ft)	Cable Length (ft)
0	0	150.0900
20	1.024	150.0927
60	9.216	150.1827

So, the change in length that the expansion loop must accommodate is 1.08 in. Assuming that there are 365 one inch temperature cycles per year and 1000 inch gust cycles per year, then the life of the loop will decrease by a factor of 0.27 as compared to its life due to temperature changes. For the 0.750 in cable mentioned above, the life of the loop should be degraded from about 49 years to about 13 years.

CONCLUSION

The basic sag and tension equations have been expanded to account for changes in temperature and load. Special configurations, such as tight lashing and "Figure 8" type cable with composite materials, were also addressed. The application of these equations can help the cable engineer respond to the local utility about maximum tension and proper clearance of aerial plant. The equations can also be applied to help understand various failure modes such as center conductor pullouts and premature expansion loop cracking.

ACKNOWLEDGEMENTS

The author would like to express his thanks to Times Fiber Communications for their support and to those in the field who, over the years, have provided him with numerous firsthand applications.

REFERENCES

[1] Edward C. Jordan, Ed. in Chief., Reference Data for Engineers: Radio, Electronics, Computer, and Communications, 7th ed. (Indianapolis: Howard Sams, 1985), pp. 48-19-20.

[2] Donald G. Fink, Ed. in Chief, H. Wayne Beaty Assoc. Ed., Standard Handbook for Electrical Engineers, 11th ed., (New York: McGraw Hill Book Co., 1978), pp. 14-64.

[3] National Electrical Safety Code, 1987 Edition. (New York: The Institute of Electrical and Electronic Engineers, Inc. 1986), pp. 256-61.

[4] Murray R. Spiegel, Mathematical Handbook of Formulas and Tables, Schaum's Outline Series (New York: McGraw Hill, 1968), p. 32.

[5] Eric Winston, "Stress Analysis of CATV Aluminum-Sheathed Cables Lashed to Hanging Steel Strand", IEEE Transactions on Cable Television, (Oct. 1976), pp. 18-24.

[6] Jack Park, The Wind Power Book, (Palo Alto, CA: Cheshire Books, 1981), p. 58, p. 164.

[7] Robert P. Lukems ed. et al, 1977 Annual Book of ASTM Standards, (Philadelphia: American Society For Testing and Materials, 1979), A475-78.

APPENDIX

TABLE 1
CABLE PROPERTIES

Unjacketed Trunk and Feeder

Type	Weight (lb/ft)	Dia. (in)	E_r (psi $\times 10^6$)	a_r (in/inF $\times 10^{-6}$)	A_r (sq in)
412	0.058	0.412	2.79	13.3	0.1333
500	0.078	0.500	2.44	13.5	0.1964
625	0.121	0.625	2.42	13.5	0.3068
750	0.171	0.750	2.37	13.5	0.4418
875	0.225	0.875	2.26	13.5	0.6013
1000	0.325	1.000	2.62	13.4	0.7854

Jacketed Trunk and Feeder

Type	Weight (lb/ft)	Dia. (in)	E_r (psi $\times 10^6$)	a_r (in/inF $\times 10^{-6}$)	A_r (sq in)
412J	0.074	0.470	2.16	13.6	0.1735
500J	0.098	0.560	1.95	13.7	0.2463
625J	0.146	0.685	2.02	13.7	0.3685
750J	0.206	0.820	1.99	13.7	0.5281
875J	0.266	0.945	1.76	13.9	0.7776
1000J	0.377	1.080	2.26	13.5	0.9161
TX565	0.106	0.625	1.77	14.0	0.3068
TX840	0.212	0.910	1.69	14.1	0.6504
TX1160	0.396	1.250	1.92	13.9	1.2282

Messengered Feeder (Figure 8)

Type	Weight (lb/ft)	Dia. (in)	E_r (psi $\times 10^6$)	a_r (in/inF $\times 10^{-6}$)	A_r (sq in)	MSGR OD (in)
412JMS	0.121	0.830	2.87	11.1	0.2207	0.109
500JMS	0.145	0.930	2.53	11.5	0.2936	0.109
625JMS	0.252	1.120	3.13	10.9	0.4272	0.186
TX565JMS	0.183	1.020	3.24	10.5	0.3657	0.186

Drop Cable and Lashing Wire

	Weight (lb/ft)	Dia. (in)	E_r (psi $\times 10^6$)	a_r (in/inF $\times 10^{-6}$)	A_r (sq in)	MSGR OD (in)
59 Quad						
Single	0.027	0.262	1.17	10.8	0.0425	
Msgr	0.039	0.417	2.03	8.89	0.0526	0.051
6 Quad						
Single	0.034	0.297	1.22	10.6	0.0556	
Msgr	0.046	0.452	1.90	9.03	0.0658	0.051
611 Quad						
Single	0.045	0.350	1.21	10.4	0.0798	
Msgr	0.066	0.527	2.24	8.66	0.0941	0.072
11 Quad						
Single	0.071	0.434	1.06	10.2	0.1267	
Msgr 1	0.100	0.657	1.91	8.59	0.1494	0.083
Msgr 2	0.096	0.658	2.60	8.44	0.1532	0.109
Lashing	0.007	0.045	---	---	---	0.109

Note: The messenger used on the 412 and 500 cable is a solid 0.109" EHS steel wire, a stranded 3/16" EHS steel wire is used on the 625 and TX565.

E_r, a_r and A_r are the resultant elastic moduli, expansion coefficients and areas respectively.

TABLE 2

MECHANICAL PROPERTIES OF SELECTED MATERIALS

Material	Elastic Modulus (psi $\times 10^6$)	Expansion Coefficient (in/inF $\times 10^{-6}$)
Steel	28-30	7.2
Copper	16	9.2
Aluminum	10	12.7
Cu Clad Al (10%byVol)	10.6	12.2
CuClad Steel (23%Cond)	24-27	7.2
Tape/Braid	4.2-6.3	11-13
Foam Diel (T4+)	0.042	78
Jacket Mat.		
LDPE	0.028	72
LLDPE	0.042	72
MDPE	0.063	83
HDPE	0.123	67
PVC	0.0011	40

TABLE 3
PROPERTIES OF
SELECTED MESSENGER WIRES
AND EHS STEEL STRANDS[7]

Type	Stranding	Dia.	WT (lb/ft)	Area (sq in)	Break Strength x 1000lb
0.051	solid	0.051	0.007	0.002043	0.200
0.072	solid	0.072	0.014	0.004072	0.365
0.083	solid	0.083	0.018	0.005411	0.485
0.109	solid	0.109	0.031	0.009331	1.800
1/8	7x0.041	0.123	0.032	0.009241	1.83
3/16	7x0.062	0.186	0.073	0.021133	3.99
7/32	7x0.072	0.216	0.098	0.028500	5.40
1/4	7x0.080	0.240	0.121	0.035185	6.65
9/32	7x0.093	0.279	0.164	0.047550	8.95
5/16	7x0.104	0.312	0.205	0.059464	11.20
3/8	7x0.120	0.360	0.273	0.079168	15.40
7/16	7x0.145	0.435	0.399	0.115590	20.80
1/2	7x0.165	0.495	0.516	0.149677	26.90

TABLE 4
NESC LOADING TABLE[3]

	Heavy	Medium	Light	Extreme Wind
Radial Ice (in)	0.5	0.25	0.0	0.0
Wind (lb/sq ft)	4	4	9	See Fig A2
Temperature(F)	0	+15	+30	+60
Wt. Adder(lb/ft)	0.30	0.20	0.05	0.0

TABLE 5
NESC CLEARANCE REQUIREMENTS[3]
60 F, No wind

Roads, alleys and other land subject to truck traffic or farm vehicles	18
Residential driveways, no truck traffic	10
Pedestrians only	8
Railroad crossings	27
Water areas sailboats prohibited (unless otherwise specified)	15

If the spans are longer than:
175 ft. in Heavy Loading Districts,
250 ft. in Medium Loading Districts, or
350 ft. in Light Loading Districts; then
additional clearance should be added with one exception. Add 0.10 ft. (0.15 ft. for railroad crossings in heavy and medium loading districts) to the clearance for every 10 ft. of span for spans longer than the ones listed above in all loading districts. Additional clearance is not necessary if the difference between the initial sag (at 60°F with no wind) and final sag at either 32°F with radial ice (from Table 4) and no wind, or 120°F with no wind (and no ice) is greater than the added clearance.

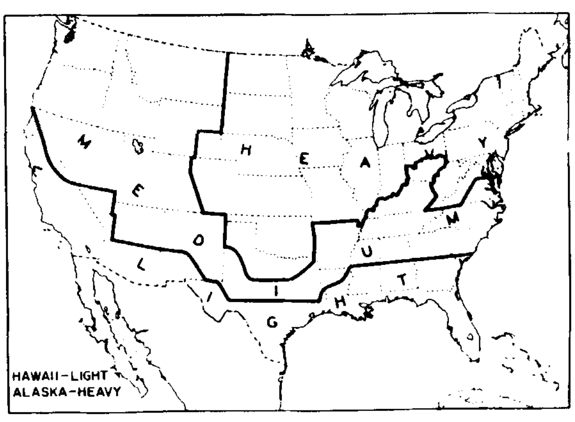

FIGURE A1
NESC LOADING DISTRICT MAP

Printed with permission.

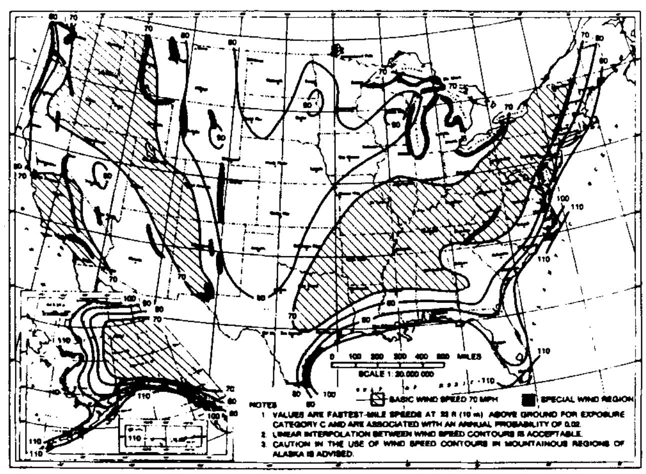

FIGURE A2
NESC EXTREME WIND LOADING MAP

Printed with permission.