EFFECTS OF CHANNEL LOADING ON COMPOSITE TRIPLE BEAT AND CROSS MODULATION

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Distortion in CATV amplifiers is due to the non-linearity of the hybrids and is influenced by channel loading and signal level. Composite triple beat and cross modulation, which are both forms of third order distortion, increase 2 dB for every 1 dB increase in signal level assuming that the hybrid is not in compression. An attempt will be made to develop an expression that accurately reflects the change in CTB and cross modulation with respect to channel loading.

The data used to generate the empirical equations for CTB and cross modulation is the result of several thousands of individual distortion measurements performed at Motorola on a total of 60 CATV hybrids using 6 different types of hybrids. The objectives of the test were to characterize the effects of distortion as a function of channel loading for sub-split, mid-split, and high-split systems.

The results of the tests show that CTB and cross modulation depend not only on the number of channels, but also on the frequency where the beat product falls. The change in CTB and the change in cross modulation due to increased channel loading cannot generally be determined by simply considering the number of new channels alone.

CASE #1 Changes in distortion caused by changes in channel loading at the high end of the band.

The equation, F (N1, N2), chosen to fit the data is an extension of the form K*log (N1/N2) that is presently used to describe changes in CTB and cross modulation as a function of new channel loading, N1, and old channel loading, N2. A correction term, G (N1, N2), is added to provide a second degree of freedom so that the least-squares error between the analytical equation and the empirical data can be further minimized. The equation

$$F(N1,N2)=K1*\log(N1/N2)+G(N1,N2) \quad (1)$$

must satisfy the following conditions:

1. As N1 approaches N2, F (N1,N2) approaches 0
2. F (N1,N2) + F (N2,N3) = F (N1,N3)
3. F (N1,N2) = -F (N2,N1)
4. F (N1,N2) must continuously increase as the number of channels increases

CTB distortion is influenced not only by the number of beats falling on the channel under test but also by the frequency of the test channel. A 234-450 MHz high-split system with 37 channels has worse distortion at 450 MHz than a 54-330 MHz sub-split system with 41 carriers does at 330 MHz. In a cable system, if the number of channels at the high end of the band were to be increased in a linear rate, the CTB would increase in a greater than linear rate. One reason for this is that the number of triple beats at any channel is proportional to the square of the number of channels; in particular, for a system with equally spaced channels, the number of triple beats at the highest channel is $[(N-1)/2]^2$. Conditions #4 above is satisfied by this relationship.

A simple equation that meets these four conditions is:

$$\Delta CTB=K1*\log(N1/N2)+K2*(N1^2-N2^2)/2000, \quad (2)$$

and

$$\Delta \text{Cross modulation} =K3*\log(N1/N2)+K4*(N1^2-N2^2)/2000 \quad (3)$$

Where ΔCTB and ΔCross modulation are the changes in distortion in going from old channel loading, N2, to new channel loading, N1.

The data used to curve fit equations (2) and (3) is the result of thousands of distortion measurements

TABLE 1

CHANNEL LOADINGS USED FOR HYBRID TESTS

SUB-SPLIT	MID-SPLIT	HIGH-SPLIT
61 channels 55.25-451.25MHz	47 channels 175.25-451.25MHz	37 channels 235.25-451.25MHz
52 channels 55.25-397.25MHz	38 channels 175.25-397.25MHz	28 channels 235.25-397.25MHz
41 channels 55.25-331.25MHz	27 channels 175.25-331.25MHz	17 channels 235-25-331.25MHz

taken on individual hybrid amplifiers. CTB and cross modulation were measured at the low end and the high end of the band for the 6MHz channel loadings listed in Table 1.

Distortion measurements were taken at 46dBmV hybrid output level with 0dB tilt. These measurements were repeated for 3dB and for 6dB of cable equivalent tilts for sub-split loading. Mid-split and high-split channel loadings were obtained by simply turning off the lower channels without readjusting the tilt as it was set up for the sub-split case. The test results showed that CTB is generally worse at the higher frequencies and cross modulation is worse at the lower frequencies. Also CTB and cross modulation did not get substantially better in going from 450MHz sub-split to 450MHz high-split loading, although they were progressively worse by comparison in going from 330MHz sub-split to 450MHz high-split channel loading.

The test data was used to solve for the coefficients K1, K2, K3, and K4 in equations (2) and (3) using a least squares solution to an overdetermined system of linear equations. These coefficients are listed in Table 2.

The mean-square error in these tables is given by

$$\sqrt{\sum_{i=1}^{N} \frac{r_i^2}{N}}$$

where r_i is the residue of the ith linear simultaneous equation used to solve for the coefficients K1, K2, K3, and K4.

The data taken at 0dB tilt, 3dB tilt, and 6dB tilt can be combined to provide a single approximating equation for these three cases without increasing the mean-square error. These coefficients are listed in Table 3.

Further reduction is possible because the coefficients K1, K2, K3, and K4 can be closely described by an exponential function. The resulting equations are:

$$\Delta CTB = 22.378 * \log(N1/N2) + 4.92 * e^{0.0029FL} * (N1^2 - N2^2)/2000 \quad (4)$$

ΔCross Modulation

$$= 43.526 * e^{0.001965FL} * \log(N1/N2) - 2.055 * e^{0.004189FL} * (N1^2 - N2^2)/2000 \quad (5)$$

Equations (4) and (5) can be used to predict the change in CTB and the change in cross modulation as channels are added (or subtracted from) the high end of the band with the low end fixed at some predetermined frequency, FL, between 54MHz and 234MHz. The computed change in distortion as a function of channel loading is listed in Table 4.

CASE #2 Changes in distortion caused by changes in channel loading at the low end of the band.

If channel loading is changed by adding (or removing) channels from the low end of the spectrum, with the high end constant, then a new set of equations must be developed to describe the change in CTB and the change in cross modulation. Applying least squares solution to an overdetermined set of linear simultaneous equations yields the coefficients listed in Table 5.

TABLE 2

ΔCTB

SUB-SPLIT	K1	K2	MEAN-SQUARE ERROR
0dB tilt	36.7895	3.16712	0.5309
3dB tilt	17.9795	6.8319	0.4994
6dB tilt	12.6939	7.67	0.6253
MID-SPLIT			
0dB tilt	27.6869	6.1294	0.5625
3dB tilt	23.6957	7.0402	0.6484
6dB tilt	14.7859	9.7131	0.8102
HIGH-SPLIT			
0dB tilt	25.9	8.6362	.7773
3dB tilt	22.396	10.222	.9463
6dB tilt	19.474	11.6365	.8086

ΔCROSS MODULATION

SUB-SPLIT	K3	K4	MEAN-SQUARE ERROR
0dB tilt	34.2969	-1.9995	0.6495
3dB tilt	37.8658	-2.3286	0.6046
6dB tilt	43.3751	-3.4727	0.5979
0dB tilt	31.8567	-3.7134	0.7525
3dB tilt	32.0924	-3.7179	0.7327
6dB tilt	33.4841	-4.9949	0.7396
0dB tilt	28.8249	-6.1806	0.8077
3dB tilt	26.6492	-5.6224	0.7234
6dB tilt	24.325	-4.9346	0.8726

The coefficients are listed for tilts of 0dB, 3dB, and 6dB. It is not possible to continue toward a single equation that applies for any value of tilt (as was done in case #1) without seriously degrading the mean-square-error. In order to apply the information in Table 5 to a system with, for example, 4dB of tilt it is necessary to compute ΔCTB and Δcross modulation assuming 3dB of tilt and 6dB of tilt and interpolate. As stated earlier, tilt is cable equivalent tilt referenced from 54MHz to the high channel (331.25MHz, 397.25MHz or 451.25 MHz) for sub-split, mid-spit and high-split systems.

CASE #3 Changes in distortion caused by a change in channel loading at the low end and at the high end, simultaneously.

Calculating changes in distortion caused by changes in channel loading at the low end and at the high end of the band is simply an application of case #1 and case #2. A small difference in the computed result will occur depending on whether the change in distortion is computed by first considering the change in channel loading at the high-end (case #1) and then be adjusting for changes in channel loading at the low-end (case #2), or vice-versa. The change in distortion should be computed both ways and the result averaged.

CASE #4 Distortion in a system spaced other than 6MHz.

Distortion in a non-6MHz system can be calculated relative to a comparable 6MHz spaced system by

$$10*\log(N1/N2) \qquad (6)$$

where N1 is the number of triple beats at frequency, F, in a 6MHz system and N2 is the number of triple beats at the same frequency, F, in a system spaced other than 6MHz. The use of the same equipment for both systems and the same high and low-end frequencies is implied. The number of triple beats and the computer program to calculate the number

TABLE 3

ΔCTB

K1	K2		MEAN-SQUARE ERROR
22.4876	5.8897	Sub-Split	0.5882
22.056	7.6275	Mid-Split	0.7096
22.59	10.165	High-Split	0.8702

Cross Modulation

K3	K4		MEAN-SQUARE ERROR
38.51	-2.6003	Sub-Split	0.6238
32.477	-4.142	Mid-Split	0.7544
26.5999	-5.579	High-Split	0.8298

Δ TABLE 4

Sub-Split (54MHz to 330/400/450MHz)

	Bandwidth Change
ΔCTB = 4.33dB	330MHz to 400MHz
ΔCTB = 4.55dB	400MHz to 450MHz
ΔXMOD = 2.64dB	330MHz to 400MHz
ΔXMOD = 1.35dB	400MHz to 450MHz

Mid-Split (174MHz to 330/400/450MHz)

	Bandwidth Change
ΔCTB = 6.00dB	330MHz to 400MHz
ΔCTB = 4.95dB	400MHz to 450MHz
ΔXMOD = 3.34dB	330MHz to 400MHz
ΔXMOD = 1.41dB	400MHz to 450MHz

High-Split (234MHZ to 330/400/450MHz)

	Bandwidth Change
ΔCTB = 7.41dB	330MHZ to 400MHz
ΔCTB = 5.71dB	400MHz to 450MHz
ΔXMOD = 4.38dB	330MHz to 400MHz
ΔXMOD = 1.58dB	400MHz to 450MHz

of triple beats occurring at any frequency for a 6MHz spaced system is given in Table 6. The computer program calculates, for each channel, all possible triple beats, FB, produced by F1, F2 and F3 such that

FB = F1 + F2 - F3
FB = F1 - F2 + F3
FB = -F1 + F2 + F3
FB = 2F1 - F2, and
FB = F2 - 2F1.

The situation where the triple beat falls on the same frequency as a carrier producing that triple beat, is not allowed.

TABLE 5

Δ CTB

Upper Frequency 450MHz	K1	K2	Mean-Square Error
0dB tilt	21.2408	-0.8260	0.1732
3dB tilt	21.6743	-1.7904	0.08844
6dB tilt	12.4627	-0.94018	0.10
400MHz			
0dB tilt	21.1186	-0.8089	0.4007
3dB tilt	19.498	-1.6228	0.2436
6dB tilt	17.6072	-2.3779	0.21505
330MHz			
0dB tilt	21.7981	-1.2693	0.3385
3dB tilt	19.6684	-2.7336	0.3498
6dB tilt	20.2212	-4.9050	0.3029

Δ Cross Modulation

Upper Frequency 450MHz	K3	K4	Mean-Square Error
0dB tilt	27.3634	-0.1026	0.38814
3dB tilt	29.2604	-1.0479	0.3388
6dB tilt	20.69899	-0.21577	0.52345
400MHz			
0dB tilt	22.686	-0.3277	0.5559
3dB tilt	23.0488	-1.332	0.5736
6dB tilt	21.74025	-1.816	0.59137
330MHz			
0dB tilt	21.1496	-0.1468	0.4165
3dB tilt	18.5581	-0.7348	0.3918
6dB tilt	17.828	-1.9623	0.4138

TABLE 6

6MHz SPACING

NUMBER OF TRIPLE BEATS PER CHANNEL

FREQ	1	2	3	4	5	6	7	8	9	10	11
55.25		640		615		435			240		
61.25		664	608	639		455			254		
67.25		687	632	661		473			267		
77.25		57	56	56		48			37		
83.25		57	56	56		48			37		
121.25		895	849	865		641			385		
127.25		921	875	890		662			401		
133.25		944	898	913		681			414		
139.25		965	920	933		697			425		
145.25		985	941	953		713			435		
151.25		1004	961	971		727			444		
157.25		1022	980	989		741			452		
163.25		1039	998	1005		753			459		
169.25		1055	1015	1021		765			465		
175.25	529	1071	1032	1036		776	342		471	169	
181.25	550	1086	1048	1051		787	359		476	180	
187.25	571	1101	1063	1065		797	375		481	191	
193.25	590	1114	1077	1078		806	390		484	200	
199.25	609	1127	1090	1090		814	404		489	209	
205.25	626	1138	1102	1101		821	417		491	216	
211.25	643	1149	1113	1111		827	429		492	224	
217.25	658	1158	1123	1120		832	440		491	228	
223.25	673	1167	1132	1128		836	450		490	233	
229.25	686	1174	1140	1135		840	459		488	236	
235.25	699	1181	1147	1141	324	844	467	182	487	239	64
241.25	710	1186	1153	1146	340	845	474	194	484	240	70
247.25	721	1191	1158	1150	356	845	480	205	481	241	76
253.25	730	1194	1162	1154	370	844	485	215	476	240	80
259.25	739	1199	1166	1158	384	842	489	224	471	241	84
265.25	746	1201	1169	1159	396	841	492	232	464	240	86
271.25	753	1202	1170	1159	408	839	494	239	457	239	88
277.25	758	1201	1170	1158	418	836	495	245	449	236	88
283.25	763	1200	1169	1156	428	832	495	250	442	233	88
289.25	766	1198	1168	1155	436	827	495	254	434	228	88
295.25	769	1197	1167	1153	444	821	495	257	426	223	88
301.25	770	1194	1165	1150	450	814	494	259	416	216	86
307.25	771	1191	1162	1146	456	806	492	260	406	209	84
313.25	770	1186	1158	1141	460	797	489	260	394	200	80
319.25	771	1181	1153	1135	464	787	485	260	382	191	76
325.25	770	1174	1147	1128	466	776	480	260	367	180	70
331.25	769	1167	1140	1120	468	764	474	259	350	169	64
337.25	766	1158	1132	1111	468	751	467	257			
343.25	763	1149	1123	1101	468	738	459	254			
349.25	758	1138	1113	1090	468	725	450	250			
355.25	753	1127	1102	1078	468	712	440	245			
361.25	746	1114	1090	1065	466	698	429	239			
367.25	739	1101	1077	1051	464	683	417	232			
373.25	730	1086	1063	1036	460	667	404	224			
379.25	721	1071	1048	1020	456	650	309	215			
385.25	710	1054	1032	1003	450	632	375	205			
391.25	699	1037	1015	986	444	612	359	194			
397.25	686	1019	997	969	436	589	342	182			
403.25	673	1002	979	952	428						
409.25	658	984	961	934	418						
415.25	643	966	942	915	408						
421.25	626	946	922	895	396						
427.25	609	926	901	874	384						
433.25	590	904	879	852	370						
439.25	571	882	856	828	356						
445.25	550	857	832	801	340						
451.25	529	830	805		324						

8MHz SPACING

FREQ	NUMBER OF TRIPLE BEATS
153.25	342
161.25	359
619.25	375
177.25	390
185.25	404
193.25	417
201.25	429
209.25	440
217.25	450
225.25	459
233.25	467
241.25	474
249.25	480
257.25	485
265.25	489
273.25	492
281.25	494
289.25	495
297.25	495
305.25	495
313.25	495
321.25	494
329.25	492
337.25	489
345.25	485
353.25	480
361.25	474
369.25	467
377.25	459
385.25	450
393.25	440
401.25	429
409.25	417
417.25	404
425.25	390
433.25	375
441.25	359
449.25	342

COLUMN DESCRIPTION

1 Number of triple beats on any channel for a 47 channel, 6MHz spaced mid-split system.

2 Number of triple beats on any channel for a 60 channel, 6MHz spaced sub-split system.

3 Number of triple beats on any channel for a 59 channel, 6MHz spaced sub-split system.

4 Number of triple beats on any channel for a 59 channel, 6MHz spaced sub-split system.

5 Number of triple beats on any channel for a 37 channel, 6MHz spaced high-split system.

6 Number of triple beats on any channel for a 52 channel, 6MHz spaced sub-split system.

7 Number of triple beats on any channel for a 38 channel, 6MHz spaced mid-split system.

8 Number of triple beats on any channel for a 28 channel, 6MHz spaced high-split system.

9 Number of triple beats on any channel for a 41 channel, 6MHz spaced sub-split system.

10 Number of triple beats on any channel for a 27 channel, 6MHz spaced mid-split system.

11 Number of triple beats on any channel for a 17 channel, 6MHz spaced high-split system.

12 Number of triple beats on any channel for a 38 channel, 8MHz spaced system.

NOTE: For an HRC system, the triple beat caused by F1+F2+F3 must also be included in the program. This can be done at line 205 with 205 count(INT(A(I)+A(J)+A(J)+A(K))=count(INT(A(I)+A(J)+A(K))+1.

```
10 DIM A(78),COUNT(1100)
20A(1)=55.25
30A(2)=61.25
40A(3)=67.25
50A(4)=77.25
60A(5)=83.25
70FORI=1TO73
80A(I+5)=121.25+(I-1)*6
90NEXT
100 INPUT "LOW FREQ CHANNEL NUMBER", LF
110 INPUT "HIGH FREQ CHANNEL NUMBER", HF
120 FOR I=LF TO HF-2
130 FOR J=I + 1 TO HF-1
1 FOR K=J+1 TO HF
1A=ABS (A(I)+A(J)-A(K))
160  IF  A=A(K)  THEN  COUNT  (INT(A))=
COUNT(INT9A))+1
170  A=ABS(A(I)-A(J)+A(K))
180   IF A=A(J) THEN COUNT (INT(A))=
COUNT(INT(A))+1
190  A=ABS (-A(I)+A(J)+A(K))
200   IF A=A(I) THEN COUNT (INT(A))=
COUNT (INT(A))+1
210  NEXT
220  NEXT
230  NEXT
240  FOR I=LF to HF-1
250  FOR J=I + 1 TO HF
260  A=ABS (2*A(I)-A(J))
270  COUNT (INT(A))=COUNT (INT(A))+1
280  A=ABS (2*A(J)-A(I))
290  COUNT (INT(A))=COUNT (INT(A))+1
300  NEXT
310  NEXT
320  FOR I=LF TO HF
330  LPRINT A(I), COUNT (INT(A(I))
340  NEXT
350  STOP
360  END
```

For a system operated at 0dB tilt, it is easy to relate the triple beat at a particular frequency to the composite triple beat (CTB) at that frequency. To show this, six hybrids were measured at 0dB tilt and 46dBmV output level for an average CTB of -59.4dB at 445.25MHz with 54MHz - 450MHz sub-split loading. The average measured triple beat was -89.75dB at 445.25MHz (adjusted for level). With 801 triple beats present at 445.25MHz, the expected CTB, based on a triple beat measurement of -89.75dB, is -60.72, compared with an actual measured CTB of -59.4dB (see Table 7).

The purpose of this test was two-fold. It shows that the magnitude of the triple beat at any particular channel is independent of the choice of frequencies, F1, F2, and F3 that produce that beat, and that the composite triple beat can be accurately related to a single triple beat measurement by equation (6). However, the magnitude of the triple beat is dependent on the frequency on which the triple beat falls, with the higher frequencies giving worst results.

NOTE: Each triple beat measurement is an average of seven individual triple beat measurements involving different combinations of frequencies, F1, F2, and F3 such that F1 + F2 - F3 = 445.25MHz. The choice of channels for F1, F2, and F3 made no difference in the magnitude of the triple beat at 445.25MHz.

The accuracy of equation (6) for predicting ΔCTB at the highest channel as a function of the change in the number of triple beats can also be compared against the data in Table 5 using a 0dB tilted system and the data in Table 7. Here Table 5 is used to predict the change in CTB at the highest channel in the band as channels are removed from the low end of the band. The data in Table 7 used with equation (6) represents a theoretical approach and gives "close" results.

As a final illustration, the CTB of a 150MHz to 450MHz system with 8MHz channel spacing and a 3dB hybrid output tilt can be determined by application of the data in Table 5 and equation (6). A starting point is required and might, for example, be the CTB of a 54MHz to 450MHz, 6MHz spaced system operating at

TABLE 7

CTB and Triple Beat Measurements on Individual Hybrid

Hybrid	CTB at 445.25MHz	Triple Beat at 445.25MHz
1	-60.5	-91.9
2	-59.9	-88.5
3	-60.8	-92.1
4	-59.4	-90.4
5	-59.3	-89.4
6	-56.6	-86.2
AVERAGE	-59.4	-89.75

NOTE: Each triple beat measurement is an average of seven individual triple beat measurements involving different combinations of frequencies, F1, F2, and F3 such that F1 + F2 - F3 = 445.25MHz. The choice of channels for F1, F2, and F3 made no difference in the magnitude of the triple beat at 445.25MHz.

TABLE 8

ΔCTB predicted by Table			ΔCTB predicted by 10*log(N1/N2)		
Test Channel	Change in loading	ΔCTB			
445.25MHz	SS to HS	3.64dB	4.1dB,	N1 = 830,	N2 = 324
397.25MHz	SS to HS	4.90dB	5.1dB,	N1 = 589,	N2 = 182
331.25MHz	SS to HS	7.45dB	7.4dB,	N1 = 350,	N2 = 64

a 3dB hybrid output tilt with a known CTB distortion. A 1 CTB of 1.19dB is calculated from Table 5 based on N1 = 52 and N2 = 61 for a 150MHz to 450MHz bandwidth, 6MHz spacing, and a 2 CTB of 2.62 dB is calcusing 10*log (N1/N2) where N1 = 625 for a 6MHz spaced system and N2 = 342 for an 8MHz spaced system. The total CTB is therefore 3.81dB better than the performance of the same equipment operating with a 6MHz channel spacing from 54MHz to 450MHz.

SUMMARY

The results presented here are optimal in the least-squares sense for CATV cable systems employing sub-split (54MHz), mid-split (174MHz), or high-split (234MHz) bandwidths with a high end frequency between 330MHz and 450MHz. The accuracy of the approximating equations, if used outside this range, should deteriorate significantly. More data is required on a wider variety of hybrids in the 54MHz to 550MHz region in order to develop a method of predicting changes in distortion versus changes in channel loading and channel spacing throughout the entire CATV band. The author would like to thank the Motorola RF & Optoelectronic Products Division in Phoenix, Arizona, for the use of their automatic test facility that made this article possible.