A THEORETICAL EXAMINATION OF THE EFFECT OF A NONLINEAR DEVICE LOCATED AT A TAP

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ABSTRACT

Intermodulation products represent a continuing concern to CATV system operators. In most systems, trunk amplifiers are designed to suppress first and second order intermodulation products. As a result, third order intermodulation products are of primary concern, especially with the increasing use of the 5-30MHz band. Due to the large number of frequencies carried by a typical CATV cable and the often indeterminate V-I characteristics of system nonlinearities, the amplitudes and frequencies of the intermodulation products are usually difficult to predict. It is advantageous, therefore, to understand the effects of nonlinearities in CATV systems and to be able to separate the contributions of amplifiers, connectors, taps, etc. in the overall frequency spectrum. In this paper, we consider, theoretically, the generation of intermodulation products at the junction of the seizure screw of the tap and the center conductor of the trunk line. In particular, the terminal response of a branched coaxial line with a nonlinear device located at the branch is discussed. In addition, the frequency spectrum present as a result of the excitation of the nonlinear device by the fundamental frequencies on the line is derived.

INTRODUCTION

The typical CATV system exists in an often harsh environment. The cable distribution system is usually an aerial array of trunk lines, amplifiers, taps, and drop cables subjected to large fluctuations in temperature, rain, ice, wind, and other damaging and corrosive elements. The electrical integrity of the system is, therefore, often difficult to maintain. Fortunately, a cause and effect relationship has been established for the majority of signal related problems. The consumer rarely fails to inform the CATV system operator of signal degradation and the cause is usually determined using a combination of on-site inspection and previous system experience. With the increasing use of the 5-30MHz band in two way systems, a more subtle but potentially harmful problem has arisen; namely, the generation of intermodulation products in the 5-30MHz band due to nonlinear junctions in the cable distribution system.

In this paper we first examine the V-I characteristics of nonlinear junctions and the generation of harmonics as a result of applying a time-harmonic excitation to the junction. Some measured data are presented and discussed in light of previous experimental work reported in the literature. The terminal response of a general transmission line is considered and equations are derived for a transmission line with a nonlinear device in series in the line. This model is extended to a branched coaxial cable with the nonlinear device located at the branch. A discussion of the meaning and application of the model follows.

FORMULATION

A junction (or device) is considered linear over a range ΔV (volts D.C.) if the corresponding current is such that Ohm's law is obeyed. Deviations from Ohm's law are indicative of a nonlinear junction. The V-I characteristic of a corroded junction formed by the seizure screw of a terminal block and the center conductor of a coaxial trunk line is shown in Fig. 1.

In order to analyze the frequency response of this junction, the V-I curve is fitted to a Taylor series given by

$$i = a_0 + a_1 v + a_2 v^2 + a_3 v^3 + a_4 v^4 + \dots \quad (1)$$

where $a_0=0$. For an ohmic junction, all coefficients a_n for $n \geq 2$ are zero. Applying a curve fitting routine to the curve in Fig. 1 yields the following coefficients.

$$a_1=0.28 \quad (2a)$$

$$a_2=0.00 \quad (2b)$$

$$a_3=-0.10 \quad (2c)$$

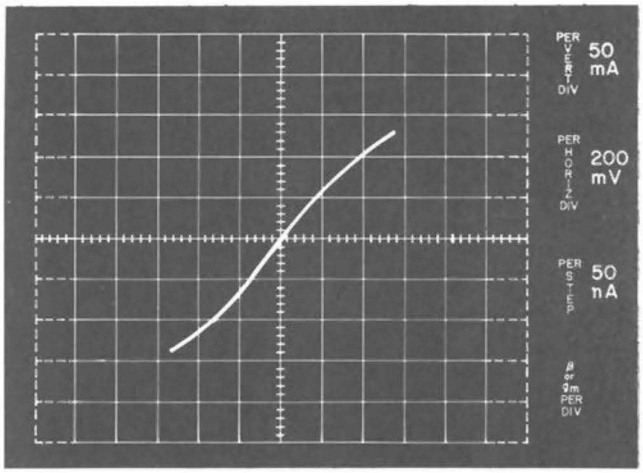

Fig. 1. Nonlinear V-I characteristic of a corroded junction.

where a_n for $n \geq 4$ are not significant. The zero value of a_2 indicates that the junction does not have any diode conduction characteristics while the value of a_3 indicates that the nonlinearity is due primarily to tunneling phenomena. This topic is discussed subsequently.

The generation of harmonics by a nonlinear junction is seen in a straightforward manner by applying a cosinusoidal excitation given by

$$v = V \cos \omega t \quad . \quad (3)$$

The general equation for the V-I characteristic of the nonlinear junction shown in Fig. 1 is given by

$$i = a_1 v + a_3 v^3 \quad (4)$$

where ω is the angular frequency ($\omega = 2\pi f$, where f is the frequency of operation).

Inserting (3) into (4) yields

$$i = a_1 V \cos \omega t + a_3 V^3 \cos^3 \omega t \quad (5)$$

and upon expanding the cube root, one obtains

$$i = a_1 V \cos \omega t + \frac{3}{4} a_3 V^3 \cos \omega t + \frac{1}{4} a_3 V^3 \cos 3\omega t . \quad (6)$$

Therefore, for an impressed voltage of angular frequency ω we obtain the fundamental ω and the third harmonic 3ω. As the number of fundamental frequencies on the line increases, the frequency spectrum, resulting from the junction's nonlinear response to the impressed voltage, becomes more complex. As an example, consider a cosinusoidal excitation with two fundamental frequencies.

$$v = V_1 \cos \omega_1 t + V_2 \cos \omega_2 t \quad (7)$$

In Table 1 the resulting frequencies and corresponding current amplitudes are shown for the excitation given by (7) impressed on the nonlinear junction represented by (3).

TABLE 1

Current Due to Voltage of Equation (7)

Frequency ($\omega = 2\pi f$)	Amplitude
ω_1	$a_1 V_1 + a_3 \left\{\frac{3}{4} V_1^3 + \frac{3}{2} V_1 V_2^2\right\}$
ω_2	$a_1 V_2 + a_3 \left\{\frac{3}{4} V_2^3 + \frac{3}{2} V_1^2 V_2\right\}$
$3\omega_1$	$\frac{1}{4} a_3 V_1^3$
$3\omega_2$	$\frac{1}{4} a_3 V_2^3$
$2\omega_1 - \omega_2$	$\frac{3}{4} a_3 V_1^2 V_2$
$2\omega_1 + \omega_2$	$\frac{3}{4} a_3 V_1^2 V_2$
$2\omega_2 - \omega_1$	$\frac{3}{4} a_3 V_1 V_2^2$
$2\omega_2 + \omega_1$	$\frac{3}{4} a_3 V_1 V_2^2$

Application of the results in Table 1 to two typical CATV channels, 2 and 6, yields the following frequency spectrum.

TABLE 2

Fundamental:	55.25MHz	
Fundamental:	83.25MHz	
Harmonics:	165.75MHz	193.75MHz
	249.75MHz	111.31MHz
	27.22MHz	221.75MHz

Clearly, for a typical CATV system with as many as 35 channels, a large number of harmonics could fall in the 5-30MHz band.

Network Equivalent Of A Transmission Line

A length ℓ of coaxial cable operating in the TEM mode can be modeled by a two-wire line equivalent and therefore a two-port equivalent network. Assume a load Z_L to exist at port 2 and a driving source voltage v to exist at port 1. The transmission-line equivalent is given in Fig. 2a. The corresponding network equivalent is shown in Fig. 2b.

The network equivalent of Fig. 2b allows one to relate V_1 and V_2 to the currents I_1 and I_2 by the impedance matrix.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (8)$$

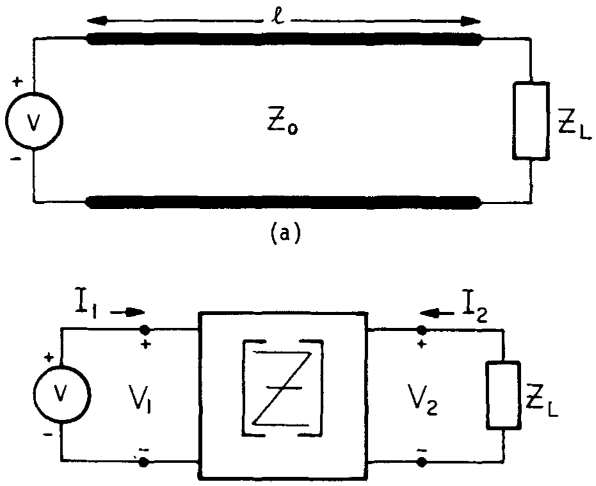

Fig. 2. (a) Two-wire line equivalent of a coaxial cable. (b) Two port network equivalent of (a).

where $V_1=v$ and $V_2=-I_2Z_L$. I_1 and I_2 are easily obtained by inverting $[Z]$ and computing $[Z]^{-1}[V]$. For reference, the matrix $[Z]$ is given by

$$[Z] = -jZ_0 \begin{bmatrix} \cot k\ell & \dfrac{1}{\sin k\ell} \\ \dfrac{1}{\sin k\ell} & \cot k\ell \end{bmatrix} \tag{9}$$

where Z_0 is the characteristic impedance of the coax (and the equivalent line). Note that the matrix elements are frequency dependent, i.e.

$$k\ell = 2\pi\left(\frac{\ell}{\lambda}\right) \tag{10}$$

where λ is the wavelength. This matrix representation is useful for computations and visualization of interactions which may take place in more complicated networks.

Now consider the simple circuit shown in Fig. 3.

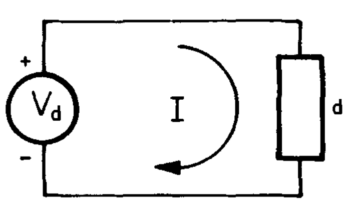

Fig. 3. Simple circuit with a nonlinear device excited by a voltage source.

The device, 'd', is the nonlinear junction considered in Fig. 1 and modeled by eq. 4 where the sign of a_3 has been explicitly indicated. Assuming

$$\left|\frac{3}{4}\, a_3\, V_d^3\right| << \left|a_1\, V_d\right| \tag{11}$$

which holds in typical practical cases, one reduces eq. 6 to

$$i \approx a_1\, V_d \cos\omega t - \frac{1}{4}\, a_3\, V_d^3 \cos 3\omega t \tag{12}$$

Now consider Fig. 4a. This circuit consists of a voltage source v, the nonlinear 'device', and a frequency dependent load $Z_L(\omega)$. The cosinusoidal excitation v is given by eq. 3. Fig. 4b is an approximate equivalent circuit with a dependent current source at frequency 3ω and with an output proportional to V_d^3.

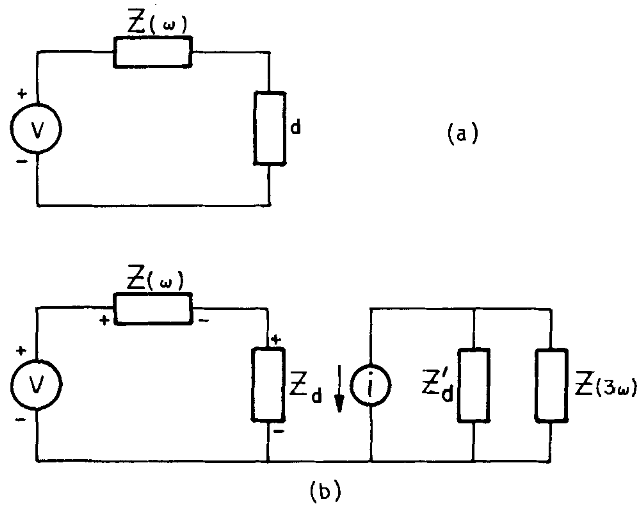

Fig. 4. (a) Simple circuit with a nonlinear device and load excited by a voltage source. (b) Equivalent circuit for (a) with dependent current source.

Z_d represents the impedance of the device due to the linear 'component' of eq. 12 ($1/a_1$). Z_d' represents the impedance of the nonlinear 'component' of eq. 12.

From Fig. 4b a voltage equivalent circuit, shown in Fig. 5, can be constructed where v_g is proportional to V_d^3. It is an approximate equivalent circuit because the dependent source V_3 is assumed to be a perturbation and I is assumed to be independent of I_3. It is possible to compute the currents I and I_3 and to arrive at the total current shown in Fig. 4b by adding the results together. The impedance Z in the right-hand side of Fig. 5 assumes its value at 3ω. If this impedance contained a contribution from a transmission line then that line would have an electrical length three times that of its value in $Z(\omega)$ in Fig. 5.

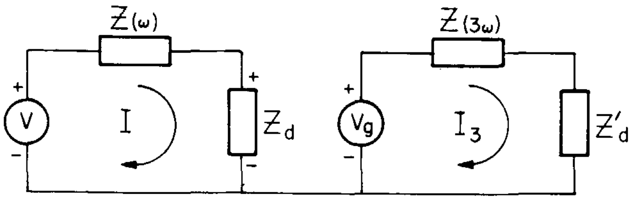

Fig. 5. Approximate equivalent circuit for Fig. 4b with dependent voltage source.

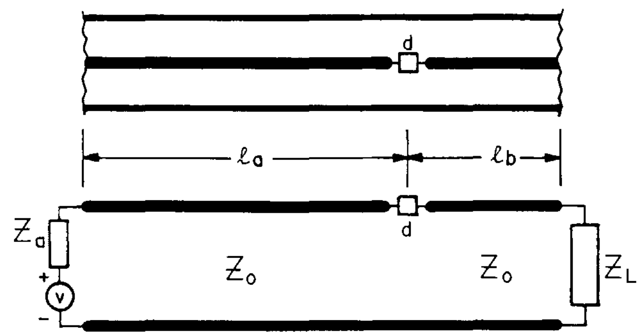

Fig. 6. (a) Coaxial line segment with a nonlinear device in series. (b) Transmission line equivalent of (a).

Network Equivalent For A Nonlinear Junction In Series With Coaxial Lines

Fig. 6a shows a segment of coaxial line with a nonlinear device in series in the line. Such a situation could arise in pin type or splice connectors where the center conductor is grabbed by the seizing mechanism. Fig. 6b is the transmission line equivalent with a voltage source v, Z_a which consists of the generator impedance and any other sections of line and load impedance Z_L. The characteristic impedance of the coax is Z_0 and the junction 'd' is located a length ℓ_a from the generator.

A network equivalent can be constructed on the basis of the concepts developed in previous sections. The network equivalent of the circuit shown in Fig. 6b is

The upper part of Fig. 7 is the network that operates at frequency ω (first harmonic). $[Z^A]$ and $[Z^B]$ are the impedance matrices for cable lengths ℓ_a and ℓ_b. All of the impedance elements must be evaluated at ω. Z_d is the first harmonic impedance of the junction. The lower part of Fig. 7 is the network that operates at frequency 3ω. The third harmonic values of the elements of the impedance matrices, $[Z^A]$ and $[Z^B]$, are now used in the calculation, Z_d' is the third harmonic impedance of the junction. Note that v does not appear in this network, since its impedance is that of a short circuit. The new (dependent) generator appears at the junction location and generates a voltage V_3 which is proportional to V_d^3. To compute, for example, the total current through the load Z_L in Fig. 6b, one first would compute the current through $Z_L(\omega)$ from the upper part of Fig.7 and then the current through Z_L (3ω) from the lower part of Fig. 7 and add the two together. Note that the voltage V_3 depends upon V_d^3 which implies that, if the input voltage v is changed, then the voltage in the lower circuit does not change proportionately. That is if v is, for example, doubled, the currents and voltages in the lower circuit will not double.

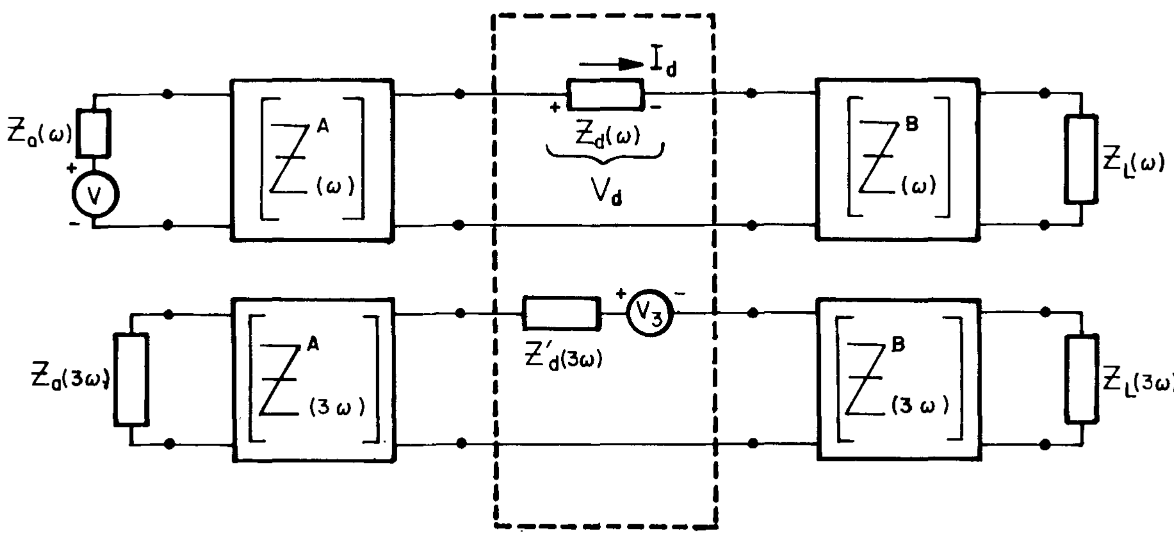

Fig. 7. Network equivalent of the circuit shown in Fig. 6b.

Network Equivalent For A Branched Transmission Line With A Nonlinear Junction At The Branch

Fig. 8a shows a branched coaxial transmission line with a nonlinear junction located at the branch. This branched structure commonly occurs at a tap. The nonlinear junction could arise due to corrosion or the presence of other thin insulating films between the trunk line center conductor and the seizure screw and terminal block. This effectively places the nonlinear junction in series with the branched line (normally a drop cable). Fig. 8b is the transmission line equivalent with a voltage source v with impedance Z_a and a load impedance Z_L. The branch is a distance ℓ_a from the generator and has a length ℓ_c with load impedandce Z_c. The nonlinear junction is labeled 'd'.

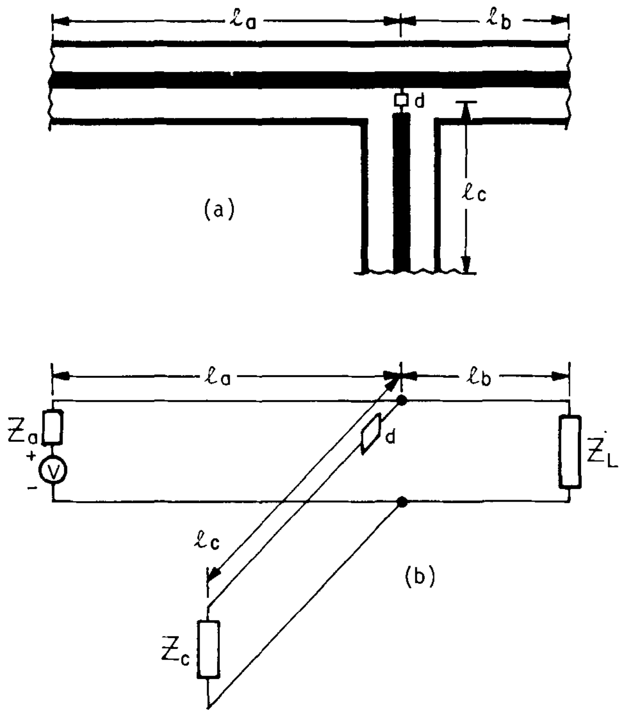

Fig. 8. (a) Branched coaxial line with a nonlinear junction at the branch. (b) Transmission line equivalent of (a).

Extending the results of the previous section, one can construct a network equivalent of the transmission line equivalent shown in Fig. 8b. This is shown in Fig. 9.

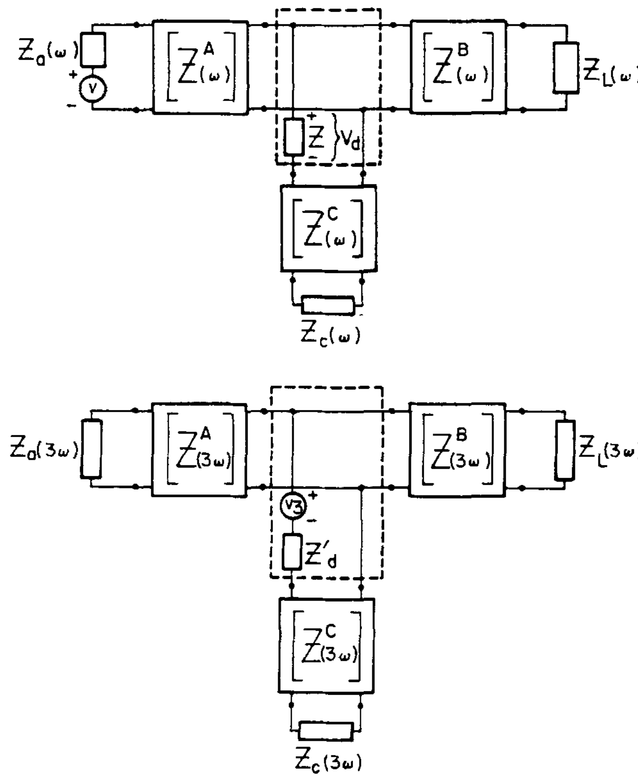

Fig. 9. Network equivalent of the circuit shown in Fig. 8b.

The concepts used in evaluating terminal currents and voltages in this network are identical to those used to evaluate similar quantities of the nonlinear junction in coaxial lines. The upper part of Fig. 9 is the first harmonic 'component' with all impedances evaluated at frequency ω. The lower part of Fig. 9 is the third harmonic 'component' with all impedances evaluated at frequency 3ω. The generator V_3, in the lower part, operates at a frequency 3ω and has an output proportional to V_d^3. The total current is the sum of the currents in the upper and lower parts.

DISCUSSION

The network equivalents presented for a junction in series in a coaxial line and a junction at a branch in a coaxial line allow one to calculate, using standard network theory, the currents and voltages present as a result of the excitation of the nonlinear junction. The junction is assumed to generate only the fundamental and the third harmonic. In actual practice, a cable system contains many 'fundamental' frequencies each of which may be modulated. This spectrum or frequencies, upon interaction with the nonlinear junction, will result in the generation of an enormous number of harmonics, even for a junction that can be described by a simple equation such as (4). For example, the excitation given by (7) would result in six additional frequencies present in the system. In the case of the network equivalent shown in Fig. 10, there would be two calculations for the upper network; one at frequency ω_2. All the impedances would assume their values at the respective generator frequency. Similarly there would be a calculation for the lower network at each of six different frequencies: $3\omega_1$, $3\omega_2$, $2\omega_1-\omega_2$, $2\omega_1+\omega_2$, $2\omega_2-\omega_1$ and $2\omega_2+\omega_1$. Each dependent generator would have a different voltage amplitude and, again, all impedances would assume their values at the respective generator frequency. To calculate the current through any particular load, one would sum the currents from these eight computations. Clearly, the extension to a more complex frequency spectrum would involve straightforward but very lengthy calculations.

The presence of the cubic term and the absence of the square term (along with terms of order higher than cubic) is given as justification for assuming that the nonlinear response of the junction is due to tunneling of electrons through an insulating film separating the metal surfaces. This observation is supported numerous times in the literature [1-5]. In general, a junction's V-I characteristic has been found to be extremely pressure sensitive. In particular see reference (5). It would not be unreasonable to envision a junction that was linear at one time of the day, becoming increasingly nonlinear as the temperature fell during the afternoon. The associated generation of harmonics would follow. Similarly, stress relaxation at metal contacts could also reduce contact pressure with the possibility of the formation of a nonlinear junction. Corrosion of any type would only aggravate the situation.

SUMMARY

In this paper the V-I characteristics of nonlinear junctions and the generation of harmonics by these junctions when a time-harmonic excitation is impressed upon them is discussed. Equivalent circuits are presented for nonlinear junctions that are in series with and at a branch between coaxial lines. From these circuits, network equivalents are constructed which allow one to calculate the terminal voltages and currents using standard network theory. It is pointed out that the nonlinear junctions typical in CATV cables and connectors are probably due to tunneling phenomena.

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