AMPLIFIER LINEARIZATION BY COMPLEMENTARY PRE OR POST DISTORTION*

A. Prochazka, P. Lancaster, R. Neumann

Delta-Benco-Cascade Ltd., Rexdale, Ontario, Canada

ABSTRACT

A technique of amplifier linearization through the use of complementary pre or post distortion correction circuits is investigated. Using the Volterra series representation distortion cancellation constraints imposed on the correction circuit are derived. A simple distortion correction circuit consisting of a diode, a resistor and a capacitor (inductor) is then devised to meet the above constraints for cancellation of the amplifier's third order distortion products.

A theoretical analysis of distortion generated in both a single stage transistor amplifier and a complementary distortion correction circuit (C.D.C.C.) is carried out to verify the realizability of this linearization technique.

* Patent Pending

INTRODUCTION

The design and manufacture of transistors and semiconductor diodes has advanced significantly since their introduction many years ago. This has resulted in the availability of semiconductor components with higher gain - bandwidth products, low noise, high power and better linearity.

Despite the advanced state of the art in the overall performance of semiconductor diodes and transistors the remaining distortion and noise are still the primary obstacles to the construction of large CATV systems. However, there are several design techniques used quite often in multistage amplifiers which help further minimize nonlinear distortion. [1] Feedback, push-pull and cascode amplifier design are the ones most common today. Others, such as feedforward and distortion compensation techniques are now being implemented.

Cascode amplifiers connected in push-pull usually suppress second order distortion far below the level considered objectionable. Also, as evidenced from theory [2]-[4] and supported by experimental observations, some partial cancellation of second order distortion along the CATV trunk always exists. The third order distortion, however, is only slightly improved in a push-pull amplifier. A significant reduction in the magnitude of the third order distortion is obtained if a cascode amplifier is used. Despite this improvement, a cascode connection is not as efficient for suppression of the third order products as push-pull is for the second order products. Also, if the amplifiers are exactly alike, crossmodulation and triple beat products f_1-f_2+f_3 will generally increase by 6 dB any time the number of amplifiers is doubled [2]-[4]. (In practice, cancellation of these products has also been observed and can be accounted for by the nonuniform magnitude and phase of the aforementioned distortion products in the amplifiers.) In conclusion, we can say that it is either crossmodulation or a build-up of triple beat products of the type f_1-f_2+f_3 which affects the performance of a vast majority of new CATV systems.

Some five years ago, a research program was undertaken by Delta-Benco-Cascade to develop a technique for a substantial reduction of crossmodulation distortion in CATV amplifiers. Out of a number of promising ideas a complementary distortion correction technique was singled out and pursued further. The scope of the research program was later extended to include a reduction of voltage additive triple beat products. This technique provides a circuit which may be connected across the signal path on either the input or the output side of an amplifier and which is designed to distort the signal in a manner complementary to the crossmodulation or triple beat distortion contributed by the amplifier itself.

MATHEMATICAL REPRESENTATION OF A CASCADE

Let us restrict our attention to the complementary post distortion correction circuit shown in Fig. 1.

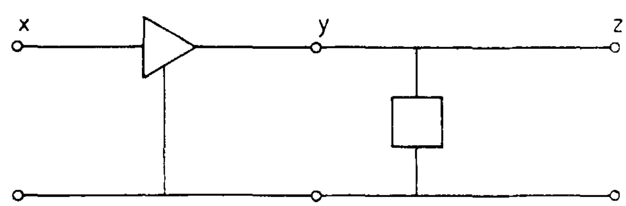

Fig. 1 · Amplifier With A Post Distortion Correction Circuit

At low frequencies, where reactive effects can be neglected the transfer characteristic of a nonlinear device, such as an amplifier, can be expressed by a power series. At high frequencies capacitive and inductive parasitics and components affect the overall performance of an amplifier. The device has a memory and can not, in general, be represented by a power series. A different analysis such as one employing the Volterra series approach, must be used. The Volterra series can account for frequency dependence and give the phase as well as the magnitude of distortion products.

For small enough signal levels and small transistor nonlinearities a third order product, such as crossmodulation or a triple beat, can be predicted accurately using a third order Volterra series. Assuming that the conditions for existence of the Volterra series are satisfied we can represent the output of a time invariant nonlinear system with memory (such as the amplifier in Fig. 1) by a third order Volterra series

$$y(t)=\sum_{n=1}^{3} \int_{-\infty}^{\infty} d\tau_1 \dots \int_{-\infty}^{\infty} d\tau_n b_n(\tau_1,\dots,\tau_n) \prod_{r=1}^{n} x(t-\tau_r) \quad (1)$$

$$b_n(\tau_1,\dots,\tau_n)=0 \text{ for } \tau_n < 0$$

where $b_1(\tau)$ is the impulse response of the amplifier. The terms in (1) are, in fact, n^{th} order convolution integrals. Each kernel b_n is assumed to be a symmetrical function of its arguments. Since the output corresponding to sinusoidal signals is of interest all computations can be carried out in the frequency domain by means of Volterra transfer functions $B_n(s_1,\dots s_n)$ defined as follows:

$$B_n(s_1,\dots,s_n)= \int_{-\infty}^{\infty} d\tau_1 \dots \int_{-\infty}^{\infty} d\tau_n b_n(\tau_1 \dots \tau_n) . \exp[-j(\omega_1\tau_1+\dots+\omega_n\tau_n)] \quad (2)$$

If B_n is known and the input consists of a linear superposition of sinusoidal signals the output can be expressed in terms of the transform B_n of the kernel b_n as will be shown later.

Similarly the output z of the post distortion correction circuit is expressed as the Volterra series of the input signal y

$$z(t)= \sum_{n=1}^{3} \int_{-\infty}^{\infty} d\tau_1 \dots \int_{-\infty}^{\infty} d\tau_n c_n(\tau_1,\dots,\tau_n) \prod_{r=1}^{n} y(t-\tau_r) \quad (3)$$

The Volterra transfer functions C_n are related to c_n according to equation (2). Both C_n and c_n are symmetric functions of their arguments. The overall first, second and third order Volterra transfer functions are given, respectively, by [5].

$$H_1(s)=B_1(s)C_1(s) \quad (4)$$

$$H_2(s_1,s_2)=B_2(s_1,s_2)C_1(s_1+s_2)+B_1(s_1)B_1(s_2).C_2(s_1,s_2) \quad (5)$$

$$H_3(s_1,s_2,s_3)=C_1(s_1+s_2+s_3)B_3(s_1,s_2,s_3)+2C_2(s_1,s_2+s_3). B_1(s_1)B_2(s_2,s_3)+C_3(s_1,s_2,s_3)B_1(s_1)B_1(s_2)B_1(s_3) \quad (6)$$

The fundamental, second and third order components at the output of the cascade can be expressed as follows

$$z_1=x_o^{(1)}|H_1(s)| \cos(\omega_1 t-\eta_1) \quad (7)$$

$$z_{12}=k_1 x_o^{(1)} x_o^{(2)} | H_2(s_1,s_2) | \cos[(\pm\omega_1 \pm\omega_2)t-\eta_2] \quad (8)$$

$$z_{123}=k_2 x_o^{(1)} x_o^{(2)} x_o^{(3)} | H_3(s_1,s_2,s_3)| . \cos[(\pm\omega_1 \pm\omega_2 \pm\omega_3)t-\eta_3] \quad (9)$$

$$s_1=\pm j\omega_1 \qquad s_2=\pm j\omega_2 \qquad s_3=\pm j\omega_3$$

with

$$x=x_o^{(1)}\cos\omega_1 t+x_o^{(2)}\cos\omega_2 t+x_o^{(3)}\cos\omega_3 t \quad (10)$$

k_1, k_2, are constants characterizing a specific type of second or third order distortion, respectively. $\eta_k(\omega)$ is the relative phase of the k^{th} order Volterra transfer function H_k at the fundamental, product frequency.

DISTORTION CANCELLATION CONSTRAINTS

By neglecting the second order interaction term in (6) and assuming the shunt impedance presented by the post distortion correction circuit is large compared to the output impedance of the amplifier we can simplify (6) to

$$H_3(s_1,s_2,s_3)=B_3(s_1,s_2,s_3)+C_3(s_1,s_2,s_3). G.\exp[-j(\rho_1+\rho_2+\rho_3)\psi -j(\pm\omega_1\pm\omega_2\pm\omega_3)t_o] \quad (11)$$

where $G=|B_1(s_1).B_1(s_2).B_1(s_3)|$ (12)

$$\rho_1=\pm 1 \text{ for } \pm\omega_1$$
$$\rho_2=\pm 1 \text{ for } \pm\omega_2 \quad (13)$$
$$\rho_3=\pm 1 \text{ for } \pm\omega_3$$

ψ is the zero frequency phase intercept of the amplifier. A constant group delay (linear phase vs frequency dependence) in the amplifier is assumed. In order to minimize a specific third order product at frequency $s=s_1+s_2+s_3$ we require

$$|H_3(s_1,s_2,s_3)|=0 \quad (14)$$

Using the substitutions

$$\left[\frac{B_3(s_1,s_2,s_3)}{C_3(s_1,s_2,s_3).G}\right]=K \quad (15)$$

$$-\gamma_3+\beta_3-(\rho_1+\rho_2+\rho_3)\psi-(\pm\omega_1\pm\omega_2\pm\omega_3)t_o=\kappa$$

we can express the magnitude of (11) as

$$|H_3(s_1,s_2,s_3)|=|C_3(s_1,s_2,s_3).G|. [(K+\cos\kappa)^2+(\sin\kappa)^2]^{1/2} \quad (16)$$

So the constraints imposed on the magnitude and phase of the third order product generated by the post distortion correction circuit are found to be

$$|B_3(s_1,s_2,s_3)|=G|C_3(s_1,s_2,s_3)| \quad (17)$$

$$-\gamma_3+\beta_3-(\rho_1+\rho_2+\rho_3)\psi-(\pm\omega_1\pm\omega_2\pm\omega_3)t_o=(2n+1)\pi \quad (18)$$

for some $n=\pm 1,\pm 2 \dots$.

For a specific type of distortion these constraints can be reduced as follows

a) vector crossmodulation distortion

The signal at the input of the amplifier is expressed as a sum of two sinusoidal signals, one of them modulated by a sine wave with a modulation frequency f_m and modulation factor m $(x_o^{(1)}=x_o^{(2)}=x_o^{(3)}=x_o)$

$$x=\tfrac{1}{2}x_o^{(1)}(1+m\cos\omega_m t)\cos\omega_1 t+x_o^{(3)}\cos\omega_3 t \quad (19)$$

It then follows (6)

$$s_1=j\omega_1 \qquad s_2=-j\omega_1 \qquad s_3=j\omega_3 \quad (20)$$

and $|B_3(j\omega_1,-j\omega_1,j\omega_3)|=G|C_3(j\omega_1,-j\omega_1,j\omega_3)|$ (21)

$$-\gamma_3+\beta_3-\psi-\omega_3 t_o=(2n+1)\pi \quad (22)$$

$\gamma_3(\omega)$, $\beta_3(\omega)$ are the relative phases of the third order Volterra transfer functions B_3,C_3, respectively, at the product frequency $\omega=\omega_1-\omega_1+\omega_3$. (Fig. 2)

b) triple beat distortion

The input signal is now given by (10). We now have

$$s_1=j\omega_1 \qquad s_2=-j\omega_2 \qquad s_3=j\omega_3 \quad (23)$$

The constraints are simplified to

$$B_3(j\omega_1,-j\omega_2,j\omega_3)=G|C_3(j\omega_1,-j\omega_2,j\omega_3)| \quad (24)$$

$$-\gamma_3+\beta_3-\psi-(\omega_1-\omega_2+\omega_3)t_o=(2n+1)\pi \quad (25)$$

where γ_3, β_3 refer now to the product frequency $\omega=\omega_1-\omega_2+\omega_3$. (Fig. 2 with $-\omega_1$ replaced by $-\omega_2$). Note that for $\omega_1=\omega_2$ these constraints are identical to (21),(22).

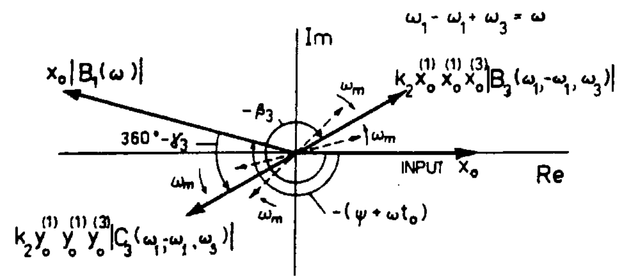

Fig. 2: Cancellation Of Vector Crossmodulation

MINIMIZATION OF AMPLITUDE CROSSMODULATION

As this high frequency distortion analysis enables one to bring into focus the phase characteristic of nonlinearities, the concept of transfer of modulation from one carrier onto another one (crossmodulation) is changed as well. Using the low frequency model (power series), only the amplitude of the originally unmodulated carrier was seen to be affected. The high frequency analysis by means of the Volterra series reveals that both the amplitude and the phase of the unmodulated carrier become modulated.[6] Let the input to the amplifier be given by (19). The total output from the amplifier at frequency ω_3 is then determined from

$$y_{\omega_3}=x_o^{(3)}|B_1(j\omega_3)|(1+m_A\cos\nu_A\cos\omega_m t).$$
$$.\cos(\omega_3 t-\beta_1+m_A\sin\nu_A\cos\omega_m t) \quad (26)$$

where

$$m_A=\frac{3m}{4}\,\frac{(x_o^{(1)})^2|B_3(j\omega_1,-j\omega_1,j\omega_3)|}{|B_1(j\omega_1)|} \quad (27)$$

is the magnitude of the vector crossmodulation (Fig. 3) and

$$\nu_A=-\beta_3+\beta_1 \quad (28)$$

where $\beta_1(\omega)$ is the linear phase delay through the amplifier at frequency ω_3 and $\beta_3(\omega)$ is the relative phase of the third order Volterra transfer function of the amplifier at product frequency $\omega=\omega_1-\omega_1+\omega_3$.

According to the NCTA Engineering Standards crossmodulation is defined as the ratio of the peak-to-peak variation of the amplitude of the test signal ω_3, due to crossmodulation, to the amplitude of the test signal with the interference signals removed. Applying this definition to equation (26) we deduce that the NCTA crossmodulation ratio is identical to twice the amplitude crossmodulation ratio

$$\text{NCTA XM} = 2M_{xm}^{(A)}=2m_A\cos\nu_A \quad (29)$$

the corresponding phase crossmodulation ratio is expressed as

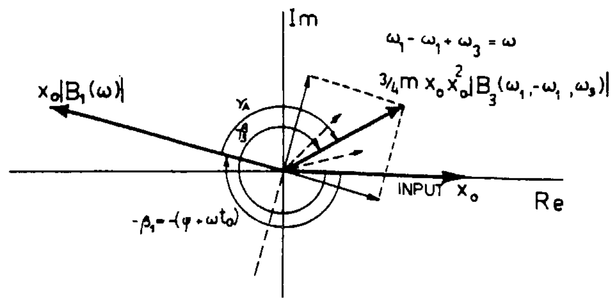

Fig. 3: Vector Representation of Crossmodulation

$$M_{xm}^{(p)}=m_A\sin\nu_A \quad (30)$$

The ratio of the amplitude to phase crossmodulation is determined solely by the phase relationship between the Volterra transfer functions of the amplifier. In practical amplifiers ν_A is determined by the transistor characteristics and by the feedback and other circuitry used. Since the amplifier design is governed by many other criterii one can hardly exercise full control of ν_A when designing an amplifier. Amplitude crossmodulation is thus likely to be inevitable in any amplifier design and is more easily minimized by the use of C.D.C.C.

Let us consider the aforementioned cascade made up of an amplifier and a post distortion correction circuit. The signal at frequency ω_3 at the output of a cascade is found to be

$$z_{\omega_3}=x_o^{(3)}|H_1(j\omega_3)|(1+m_c\cos\nu_c\cos\omega_m t).$$
$$\cos(\omega_3 t-\eta_1+m_c\sin\nu_c\cos\omega_m t) \quad (31)$$

$$m_c=\frac{3m}{4}\,\frac{(x_o^{(1)})^2|H_3(j\omega_1,-j\omega_1,j\omega_3)|}{|H_1(j\omega_3)|} \quad (32)$$

In order to minimize amplitude crossmodulation we must have

$$\nu_c=-\eta_3+\eta_1=\frac{\pi}{2}+n\pi \quad (33)$$

after substitution for η_1 from

$$\eta_1=\psi+\omega_3 t_o \quad (34)$$

and for η_3 from (11) we arrive at

$$\nu_c=\tan^{-1}\left[\frac{K\sin(\psi+\omega_3 t_o-\beta_3)-\sin\gamma_3}{K\cos(\psi+\omega_3 t_o-\beta_3)+\cos\gamma_3}\right]+r\pi \quad r=0,\pm1,\ldots \quad (35)$$

It can be shown that ν_c will attain a value of $\frac{\pi}{2}+n\pi$ provided that the denominator

$$K\cos(\psi+\omega_3 t_o-\beta_3)+\cos\gamma_3=0 \quad (36)$$

For a given amplifier many combinations of K and γ_3 will satisfy this condition. As an example let us assume that K=1. The relative phase of the vector crossmodulation generated by the correction circuit must be 180° out of phase with the vector crossmodulation generated in an amplifier, ie:

$$\gamma_3-\beta_3+\psi+\omega_3 t_o=\pi+2n\pi \quad (37)$$

This condition is basically the same as that given by (22). It is obvious that if the vector crossmodulation at the output of a cascade vanishes so does the amplitude crossmodulation (Fig. 2).

The other example we are going to mention is the situation where $\gamma_3=0$. It then turns out from (36) that the projection of the vector crossmodulation generated in an amplifier must be equal to the magnitude of the vector crossmodulation generated in a post distortion correction circuit (Fig. 4).

$$|B_3(j\omega_1,-j\omega_1,j\omega_3)|\cos(\psi+\omega_3 t_o-\beta_3)= -|C_3(j\omega_1,-j\omega_1,j\omega_3).G| \quad (38)$$

Note here that signals producing the crossmodulation distortion in a post distortion correction circuit are first amplified before being fed to the correction circuit.

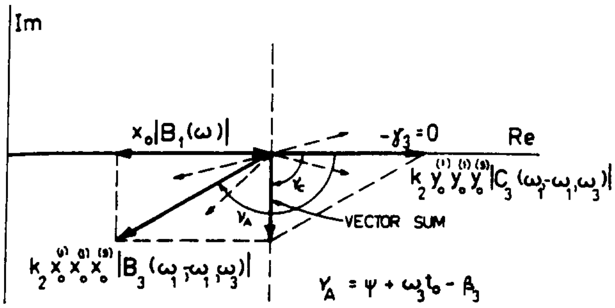

Fig. 4: Minimization of Amplitude Crossmodulation

COMPLEMENTARY DISTORTION CORRECTION CIRCUIT

The following complementary distortion correction circuit (Fig. 5) was devised to cancel amplitude crossmodulation and triple beat distortion generated in the nonlinear amplifier. It is made up of a semiconductor diode in series with an external resistor R_S' plus the biasing circuitry (not shown). The magnitude of this series external resistance must be sufficiently high so as not to appreciably affect the gain of the amplifier. From the aspect of operating point adjustment it has been found advantageous to use a Schottky or hot carrier diode with a very low junction capacitance. Usually, a reactive element in either series or parallel to the diode is needed when elimination of triple beat distortion is desired.

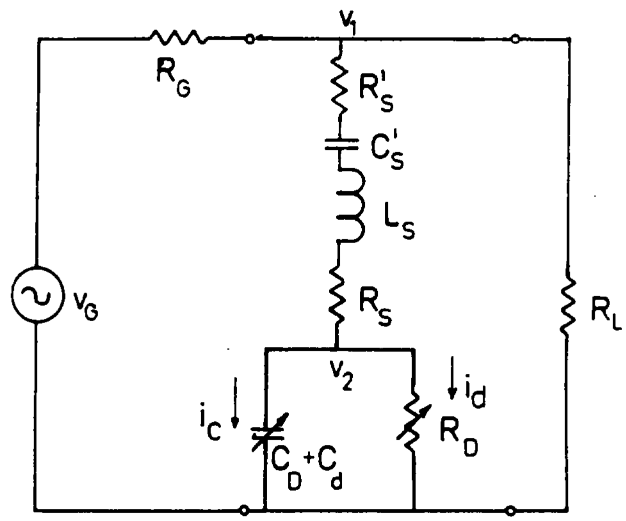

Fig. 5: Equivalent Circuit of C.D.C.C.

For purposes of analysis, the diode is represented by its small signal equivalent circuit consisting of the nonlinear junction conductance, nonlinear junction capacitance, series bulk resistance and series lead inductance. An extensive analysis of nonlinear distortion produced in a Schottky diode in series with an external impedance has been carried out and published in the literature [7]. We shall briefy summarize some results of that report. The nonlinear currents i_d and i_c are each first expressed as a Taylor series in v_2, the junction voltage. This is accomplished by the expansion of

$$I_D+i_d = I_o[\exp(a(V_D+v_d))-1] \quad (39)$$

$$i_c = \frac{dq(v_2)}{dt} \quad (40)$$

about the DC bias. The output voltage $v_1=z$ is then expressed as a Volterra series expansion of the generator voltage v_G (equal to $2y_1$ when $R_L=R_G$, $R_S' >> R_G$) given by

$$v_1=C_1(s)*v_G+C_2(s_1,s_2)*v_G^2+C_3(s_1,s_2,s_3)*v_G^3 \quad (41)$$

where * denotes an operator. By applying Kirchoff's Law, representing the impedances by their transforms and collecting all the terms of each degree we can successively find the Volterra transfer functions $C_1(s)$, $C_2(s_1,s_2)$, $C_3(s_1,s_2,s_3)$. [7]

A time sharing computer was used to compute the sum and difference intermodulation distortion and triple beat distortion. The computed and measured values of second order sum beat distortion at 266.5 MHz (Channel 13 video + Channel 2 video) are shown in Fig. 6. The magnitude of this distortion product in dB was measured using a Dix Hills Electronics SX-16 Frequency Source and R-12 Distortion Analyzer. The output level of each carrier was set at 34 dBmV. Figure 7 summarizes the computed and measured values of single channel amplitude crossmodulation. The computed and measured values of triple beat distortion at 55.25 MHz (Channel 12 video - Channel 13 video + Channel 3 video) at two different output levels are shown in Fig. 8. As can be seen there is a significant difference between the measured and computed values at an output level of 34 dBmV and at low bias. This is probably due to the higher order terms (7th, 9th) and a package capacitance which were neglected in our analysis. However, when using the C.D.C.C. to cancel the third order distortion of the amplifier, the direct current values of more than .1mA are selected. Specifically, when the DC current reaches lmA, the fifth and higher order contributions are negligible for signal levels below 50 dBmV. The following values were used for calculations: $R_S'=1000\Omega$, C_D varying from 1.8pF to 3pF at lmA, $a=37.35$, $R_G=R_L=75\Omega$.

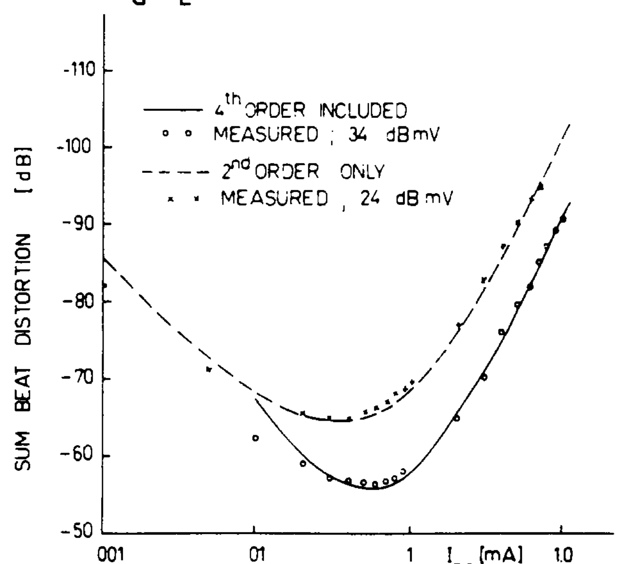

Fig. 6 Sum Beat Distortion at 266.5 MHz vs I_{DC}

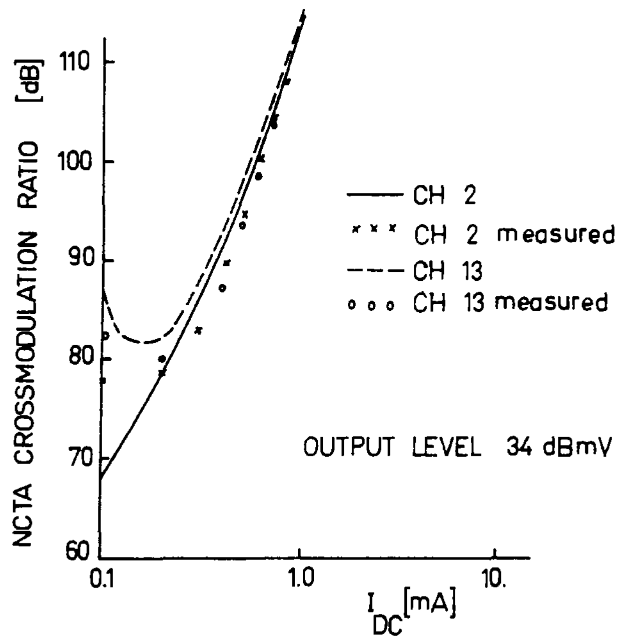

Fig. 7: Single Channel NCTA XM Ratio vs I_{DC}

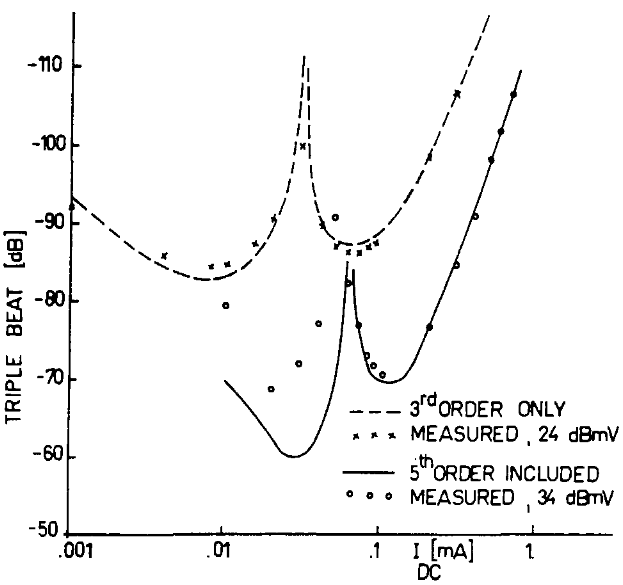

Fig. 8: Triple Beat Distortion at 55.25 MHz vs I_{DC}

As mentioned above, at higher bias currents, when the generated third order product is practically in phase with the fundamental signal, the fullfilment of the phase constraints (22), (38) is accomplished by means of an external reactive impedance. Calculated as well as measured results indicate that the magnitude of the triple beat or vector crossmodulation is only slightly changed when a 5-10pF capacitor is connected in series with the diode and a 1000Ω resistor. Such a capacitor, however, produces a sufficient change in the phase of the third order product generated in the C.D.C.C. so as to cancel a specific third order product generated in most broadband amplifiers. Sometimes, the capacitor must be replaced by an inductor in order to eliminate a specific third order product produced in a nonlinear amplifier. Cancellation of amplitude crossmodulation can be obtained with or without an external reactive element (see Section on Amplitude Crossmodulation).

DISTORTION PRODUCED IN A NONLINEAR AMPLIFIER

Let us consider a nonlinear amplifier as shown in Fig. 9. Derivation of closed form expressions for the second and third order Volterra transfer functions is an extremely difficult if not impossible task. If, however, the intrinsic parameters of a transistor are known these Volterra transfer functions can be evaluated with the help of a computer. As the phase of third order distortion products was of utmost importance to us we decided to computer-analyze distortion generated in a wideband amplifier.

A low distortion amplifier, incorporating both emitter and collector feedback, was built using a 500mW transistor having a gain bandwidth product of 5 GHz (Fig. 9). A thorough knowledge of the distortion processes in this amplifier provides insight into methods for reducing the amplifier distortion by means of adjusting the bias and the component values, and in particular, by the use of C.D.C.C. The latter method requires that both magnitude and phase of the distortion products be known so that the cancellation conditions (17), (18) can be applied. Just as in the case of the C.D.C.C., such magnitude and phase information were obtained by a Volterra series analysis of the transistor amplifier.

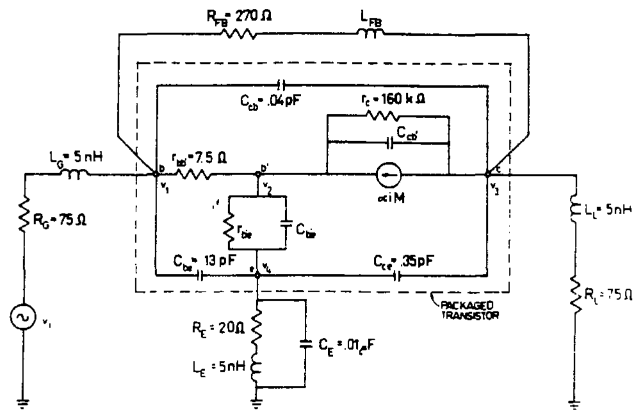

Fig. 9: Nonlinear T Model of Transistor Amplifier

The transistor model and the method of circuit analysis follow the case of Narayanan [5]. Fig. 9 shows the nonlinear T-model of the transistor with the addition of the collector-base feedback elements and a fourth node for the emitter feedback components. The nonlinearities included in the model are: base emitter nonlinear resistance and capacitance, avalanche and gain nonlinearity and collector-base nonlinear capacitance. The last nonlinearity was found to contribute negligibly to overall amplifier distortion and will henceforth be ignored.

A request for assistance in transistor modelling to several major manufacturers of CATV devices was met only by Philips who have been very co-operative in this matter. Using instruments such as Tektronix Curve Tracer 576, Boonton Capacitive Bridge 250A, Rohde & Schwarz network analyzer ZWA, we have obtained the basic model parameters for two Philips' transistors built around the same chip. The remaining parameters such as $r_{bb'}$, r_c were supplied by Philips, Eindhoven. For actual calculation, the device with a 500mW power dissipation (stripline package) was selected. The nonlinear parameters on the other hand, had to be measured individually, as they vary from transistor to transistor, particularly in the case of the gain and avalanche nonlinearities. Accurate measurements of the transistor's nonlinear characteristics yield data which can be fitted to the theoretical expressions applicable to each nonlinearity. The nonlinear resistance $r_{b'e}$ can be obtained from the base emitter I-V characteristic.

$$I = I_o \left[e^{\frac{qV_{BE}}{nkT}} - 1 \right] \qquad (42)$$

with I_o=.36mA, n=1.35 as fitted values. The nonlinear capacitance, $C_{b'e}$ consists of two components: a depletion capacitance, C_d, measured with the base emitter junction reverse biased

$$C_d = \frac{C_1}{\left(1 - \frac{V_{BE}}{Vo}\right)^{1/3}} \qquad (43)$$

and a diffusion capacitance C_D obtained from forward bias measurements.

$$C_D = C_2 \cdot I_E(V_{BE}) \qquad (44)$$

Both components are expressed here as functions of V_{BE}. Fitting of measured values gave C_1= 4.3pF, V_o= .8V, C_2= 650pF/A. Differentiation of the above expressions yields the higher order terms required for the second and third order Volterra series. Measurements of low frequency gain vs collector current were fitted by a simple polynomial which was then manipulated to give values for the nonlinear parameter α and its derivatives. The avalanche nonlinearity was determined by measuring I_C vs V_{CE}, then taking the ratio of the curved I_C characteristic to the straight line I_C characteristic which would have been obtained in the absence of avalanche multiplication, and fitting it to the equation

$$M = \frac{1}{\left[1 - \frac{V_{CB}}{V_{CBO}}\right]^n} \qquad (45)$$

with resulting values of V_{CBO}= 26V, n= 4. Derivatives of this last equation combined with the previously determined nonlinear parameters and their derivatives, plus the linear transistor parameters, complete the nonlinear T-model of Fig. 9.

A complete program was written to analyze this T-model, in a manner similar to the analysis of the C.D.C.C. mentioned above. The calculations yield the voltages of each of the circuit nodes as a Volterra series of the input voltage v_i. With v_i consisting of either 2 or 3 input signals, 2nd and 3rd order distortion, respectively (Intermodulation, Crossmodulation, Triple Beat) can be calculated. All transistor parameters can be varied in value as desired and each of the nonlinear elements can be turned 'on' or 'off' to determine the contribution of each individual nonlinearity to the overall amplifier distortion. Equally important are the calculated values for the phases of the distortion products - required information for the design of the C.D.C.C. according to the constraints given by (17), (18).

The theoretical distortion levels are obtained by calculating the appropriate 2nd or 3rd order distortion product at the desired frequency and comparing it to the linear output which the amplifier would produce at the same frequency. For example, a triple beat is calculated as the ratio, in dB, of the 3rd order term of the Volterra series of the output voltage at product frequency f_1-f_2+f_3 to the linear (Ist order term) output at frequency f=f_1-f_2+f_3. Fig. 10 shows representative calculated and measured values of triple beat distortion at Ch 2, 13 vs collector current I_C. The calculated results show a definite minimum in distortion at about 50mA bias. This minimum, or 'troughing' may be explained by careful consideration of the individual distortion contributions of the various nonlinear transistor parameters as shown in Fig. 11. The phase of each distortion component, approximately

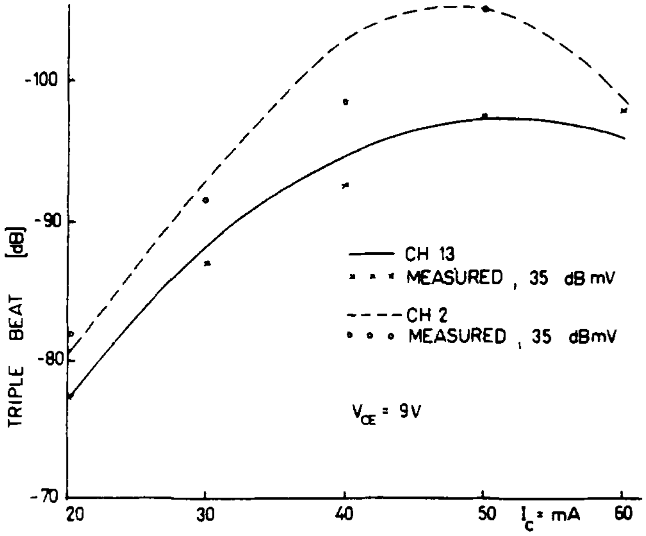

Fig. 10 Triple Beat Distortion of Amplifier

constant over the range of bias current, shows that the $r_{b'e}$ and gain components, similar in magnitude, are nearly in phase, while the avalanche component is about opposite in phase to the latter two. It is thus to be expected that, at some values of collector current, there will be a considerable degree of distortion cancellation. The depth of this minimum and its exact location depend on many factors. Where the dominant components cancel to a large degree, the residual distortion is determined by those factors which were previously neglected, the capacitive nonlinearities and higher order distortion components (5th, 7th order, etc.) due to the higher order terms in the Volterra series. The depth and location of the minimum are as well both sensitive to the exact values of magnitude and phase of the distortion components, which in turn depend on the accuracy with which the nonlinearities of this particular transistor could be determined. Considerable variation in, or even the complete absence of the distortion minimum could be expected due to variations from transistor to transistor. The experimental results of Fig. 10 indicate a distortion minimum occuring at a collector current of about 60mA, somewhat higher than the theoretical value. Agreement in magnitude between theory and experiments is close enough that theoretical values for the phase of the distortion products can be used in the equations for the C.D.C.C. Only near the distortion minimum do phase values vary greatly, undergoing a nominal 180° change.

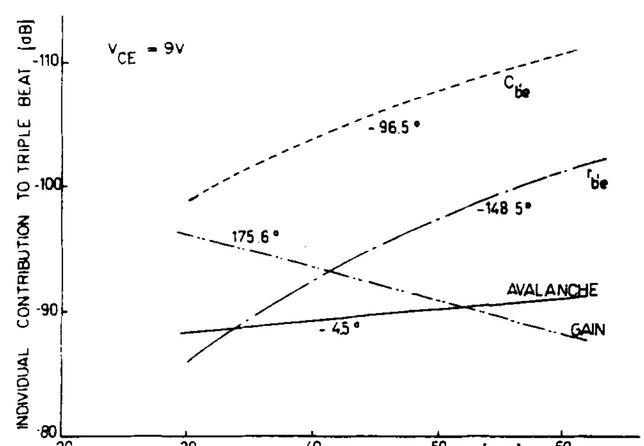

Fig. 11 Contributions of Individual Transistor Nonlinearities to Triple Beat Distortion

Theoretical values for the phase and amplitude crossmodulation can be obtained from the triple beat calculations according to (29), (30) for $-\omega_2 \rightarrow -\omega_1$. Fig. 12 compares the measured and computed values of amplitude crossmodulation.

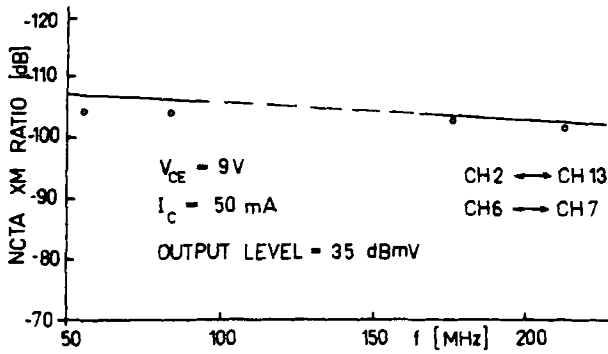

Fig. 12: Single Channel NCTA XM Ratio vs Frequency

DISTORTION CANCELLATION RESULTS

a) Narrowband Distortion Cancellation

In order to obtain perfect cancellation of a particular triple beat or vector crossmodulation distortion generated by the nonlinear amplifier, one must voltage add another product of the same frequency, magnitude and opposite phase. This additional product is produced by a complementary post distortion correction circuit. The same three signals which give rise to a triple beat or vector crossmodulation in the amplifier are, after passing through the amplifier, fed to the C.D.C.C. This circuit will then produce, due to the second and third order terms of the diode's nonlinear V-I characteristic, a distortion product at the same frequency. The magnitude of this product produced in the correction circuit is controlled by an external resistance R_S' in series with the diode and by adjustments of the DC bias. The relative phase of this distortion product is controlled by either a capacitor or inductor in series with the diode or in parallel to it. Other arrangements for controlling the magnitude and relative phase of this product have also been tried and found effective.

Let us turn our attention back to the amplifier shown in Fig. 9 with the C.D.C.C. of Fig. 5 connected at the output. Assume first that the diode is biased at $I_{DC} \approx$ lmA. The situation could now be represented as shown in Fig. 4 where $-\omega_1$ is replaced by $-\omega_2$ if the triple beat distortion is considered. If the resistance $R_S' \rightarrow \infty$ the magnitude $k_2 y_o^{(1)} y_o^{(2)} y_o^{(3)} | C_3(j\omega_1, -j\omega_2, j\omega_3)$ approaches zero and the third order distortion product generated in the amplifier is not affected. If the resistance R_S' is reduced the overall triple beat or vector crossmodulation of this cascade will start to decrease. The minimum value is attained when the vector representing the triple beat or crossmodulation distortion of this cascade is normal to the vector representing the amplifier fundamental output signal at the product frequency. This improvement in the triple beat or vector crossmodulation is given by $-20 \log | \sin\nu_A |$.

Table I Triple Beat of Amplifier

V_{CE}=9V., I_C=40mA 35 dBmV	Ch 2 (2,3,12,13)	Ch 6 (5,6,7,8)	Ch 7 (5,6,7,8)	Ch 13 (2,3,12,13)
Amplifier Without C.D.C.C. (dB)	-[illegible].5	-97.5	-94.5	-91.5
Improvement With C.D.C.C. (dB)	>20	19	6	4.5 dB
R_S' (Ω)	700 Ω	680 Ω	640 Ω	610 Ω

Table I summarizes the measured values of the triple beat improvement. The improvement in vector crossmodulation could not be measured using the Dix Hills distortion set-up as it yields only amplitude crossmodulation. However, some confirmation was obtained at higher output levels when a spectrum analyzer could be used. Table I also gives the value of the series resistance R_S' measured when the minimum was obtained. Fig. 4 indicates that this null is quite broad and so practically the same results could be obtained with a value R_S' set somewhere midway between 700 ohms and 610 ohms. Fig. 13 gives the magnitude of the triple beat product produced in the diode circuit for a given value of R_S'. Comparison of calculated values of R_S' with those needed for this partial cancellation shows good agreement.

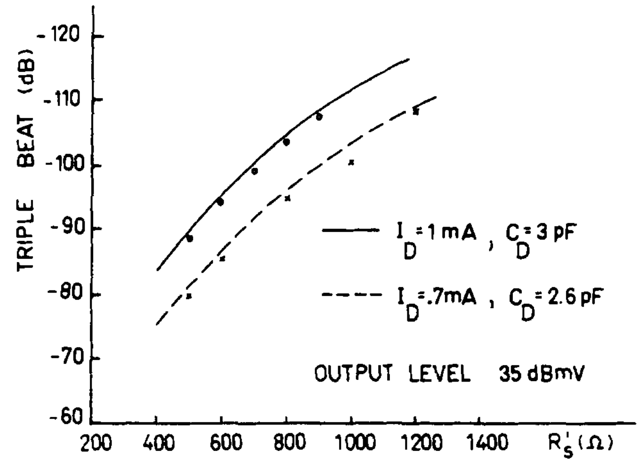

Fig. 13: Triple Beat Distortion vs R_S'

The capacitor C_S' = 1500pF is now replaced with a variable 2-18pF capacitor. The triple beat or vector crossmodulation generated by the correction circuit must now attain a relative phase of $180 - |\nu_A|$ (see Fig. 4) with respect to the horizontal axis in order to achieve a complete cancellation. Calculations have indicated that C_S' of 10pF results in $-\gamma_3 = -330^o$. This was verified experimentally when a value of C_S' from 7-10pF was needed to arrive at perfect cancellation of triple beat and vector crossmodulation distortion on Ch 2 and Ch 6. Since the magnitude of the third order distortion product produced in the correction circuit starts decreasing slightly for C_S' below 10pF the value of the resistor R_S' has to be lowered by about 10%. A slight further decrease in R_S' (compared to partial cancellation as mentioned above) was necessary since now the opposing vectors have equal magnitudes. Cancellation of distortion products on Ch 7 and Ch 13 was obtained with values of C_S' around 5pF.

As indicated by the computed results complete cancellation for one particular frequency can also be obtained with C_S' = 1500pF and an inductor of 200nH in parallel with the diode. In practice, much higher values of inductance had to be used to compensate for the intrinsic diode capacitance. Note again that inclusion of any reactive element in the C.D.C.C. makes this circuit effective in a narrow band of frequencies only.

When a value of R_S' much below 1000 ohms is needed the amplifier gain is slightly affected. This can be avoided by changing the bias current, connecting two diodes in series etc. However, DC bias of lmA for the diode was generally used when amplifiers built with Philips, TRW, or MSC stud devices were linearized and so it was used here as well. In practice, ¼ dB change in amplifier gain can be tolerated.

b) Broadband Distortion Cancellation

Broadband cancellation of the amplifier triple beat and vector crossmodulation products through the use of a simple C.D.C.C. (Fig. 5) is possible provided that the magnitude of these products is approximately constant and $|\nu_A| \approx 180^o$ or 0^o over the frequency range of interest. Amplifiers incorporating transistors with a high gain bandwidth product f_T often come close to meeting these two conditions. In such a situation, the parameters of the C.D.C.C. are adjusted for a complete cancellation of the triple beat and vector crossmodulation distortion at the high frequency end (amplifier triple beat usually increases with frequency) with only a partial cancellation of these products at the low frequency end.

We have verified both theoretically and experimentally [1] that the level of distortion noise due to a large number of triple beat products generated in a typical CATV trunk amplifier is much higher in a channel near the 300 MHz end. Using the simple C.D.C.C. of Fig. 5 we can reduce the level of this distortion noise by 6–8 dB at the high frequency channel while the level of distortion noise at the low frequency channel is improved by 1–2 dB. The fact that a complete elimination of this composite triple beat can not be realized may be accounted for by the higher order terms of the nonlinear transfer characteristics and as well by the dependence of the Volterra transfer functions on the input frequencies.

The amplifier second order distortion is not affected because the level of the second order distortion produced in the C.D.C.C. is much lower (see Fig. 6 at DC bias of 1mA). If, however, the second order distortion of the amplifier were degraded by the C.D.C.C. one could employ two diodes in a push-pull arrangement to suppress the second order distortion from the C.D.C.C.

In a more general case, the magnitude of the triple beat and vector crossmodulation distortion generated in the amplifier increases with frequency and the relative phase $|\nu_A|$ varies monotonically over the frequency range of interest. Such was the case with our model amplifier of Fig. 9. It was found that a broadband cancellation of the triple beat and vector crossmodulation products was possible provided that a parallel resonant circuit was incorporated in the design of the C.D.C.C.

c) Amplitude Crossmodulation Minimization

The vector representation of amplitude crossmodulation minimization is shown in Fig. 4. In order to eliminate amplitude crossmodulation produced in a nonlinear amplifier the post distortion correction circuit must produce amplitude crossmodulation of the same magnitude. The magnitudes of the corresponding vector crossmodulations may however differ. Amplitude crossmodulation of a cascade comprised of a nonlinear amplifier and a post distortion correction circuit (Fig. 4) vanishes as soon as the vector sum of

$k_2 y_o^{(1)} y_o^{(2)} y_o^{(3)} C_3(j\omega_1,-j\omega_1,j\omega_3)$ and

$k_2 x_o^{(1)} x_o^{(2)} x_o^{(3)} B_3(j\omega_1,-j\omega_1,j\omega_3)$ is normal to the

vector $x_o^{(3)} B_1(\omega_3)$. The magnitude of the overall vector crossmodulation reaches a minimum when amplitude crossmodulation vanishes (all amplitude crossmodulation converted into phase crossmodulation).

The experimental investigation has confirmed that amplitude crossmodulation can be made to vanish at any product frequency by properly adjusting the DC bias and the resistor R_S in series with the diode. Table II summarizes the measured values of the NCTA crossmodulation ratio of the amplifier shown in Fig. 9. The measurement was done with eleven amplitude modulated carriers whose peak levels at the modulation crest were equal to the peak level of the test (unmodulated) carrier. Reductions of the NCTA crossmodulation ratio of more than 30 dB at an output level of 35 dBmV could be observed on the distortion analyzer. Actual improvements were much higher but the distortion analyzer can only measure the distortion product magnitude down to about -120 dB. When the amplifier output level was increased to 45 dBmV, improvements of 50 dB could be observed (NCTA crossmodulation ratio reduced from -70 dB down to less than -120 dB).

Table II gives the measured values of the NCTA crossmodulation ratio before the C.D.C.C. was connected at the output and the values of the resistor R_S' needed to cause the NCTA crossmodulation ratio of this cascade to vanish. If we relate these values of R_S' to the NCTA crossmodulation ratio according to Fig. 7 (note that the values given in Fig. 7 must be degraded by approximately 20 log 11 to sum up contributions from all amplitude modulated carriers) we find good agreement between the measured values of the NCTA crossmodulation ratio of the amplifier and the C.D.C.C. All the above tests were done with the diode in the C.D.C.C. biased at ImA and C_S' = 1500pF). Similar results could be achieved at a different DC bias (ie: I_{DC}=.7mA).

Table II. 12 Channel NCTA XM Ratio of Amplifier

V_{CE}=9V, I_C = 50mA 35 dBmV	Ch 2	Ch 6	Ch 7	Ch 13
Amplifier Without C.D.C.C (dB)	-83	-82.5	-81.5	-82
R_S' (Ω)	770	790	800	820
Measured Improvement (dB)	16	18	22	18
Calculated Improvement (db)	18.1	20.8	31	24.5
Measured Improvement With C_S' = 7pF (dB)	20.5	22.5	21.5	23.5

It should be understood that narrow band amplitude crossmodulation cancellation is also possible at much lower values of diode bias current even though the relative phase $-\gamma_3$ of the vector representing the crossmodulation generated in the C.D.C.C. is not equal to zero (see equation (36)). Minimization could also be achieved if we used other values of series or parallel capacitance (inductance). As mentioned above amplitude crossmodulation vanishes if the vector crossmodulation is cancelled (Fig. 2). Note also that minimization of amplitude crossmodulation is accompanied by a partial cancellation of the triple beat distortion at the same product frequency (see Fig. 4 with $-\omega_1$ replaced by $-\omega_2$).

Theoretical as well as experimental investigations indicate that amplitude crossmodulation in wideband amplifiers varies with frequency. Under such conditions a complete elimination of amplitude crossmodulation over the frequency band of interest by means of a simple C.D.C.C. is not possible. The amount of broadband improvement (partial reduction) is related to the actual difference in measured amplitude crossmodulation at different product frequencies. With capacitor C_S' set to 1500pF, a minimum broadband improvement (Ch 2 through Ch 13) of 16 dB could be obtained with our model amplifier of Fig. 9. Expected as well as measured improvements in the NCTA crossmodulation ratio on four tested carriers at one particular value of the linearizing resistor R_S' = 790 ohms are given in Table II.

A broadband improvement of more than 20 dB in 12 channel NCTA crossmodulation ratio was attained with C_S' set to 7pF (see Fig. 14). If desired, even better results can be achieved if a resonant circuit is incorporated in the C.D.C.C.

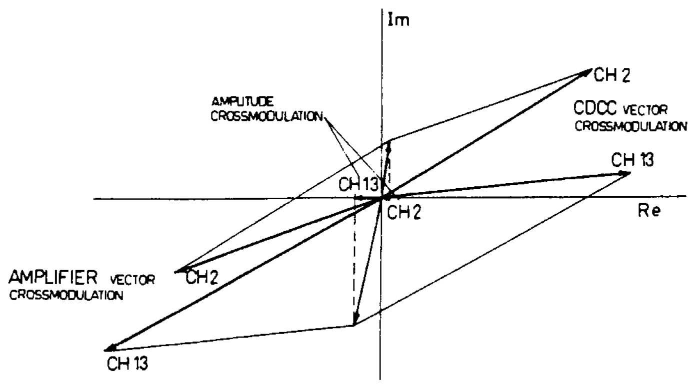

Fig. 14 Broadband Minimization of NCTA XM Ratio

ACKNOWLEDGEMENT

The authors wish to thank J. Greenan for her excellent job in preparing the paper and J. Prochazka for drawing the figures.

REFERENCES

[1] A. Prochazka, Improving CATV system performance through amplifier design, CCTA Convention,18th, Technical Records, Vancouver 1975

[2] A. Prochazka, Cascading of distortion in CATV trunk line, IEEE Trans. Broadcasting, Vol BC-20, pp 25-32, June 1974

[3] A. Prochazka, P. Lancaster, Limitations in distribution of TV signals over cable transmission systems, Technical Report, Delta-Benco-Cascade Ltd., December 1974

[4] R.J. Seacombe, A. Prochazka, P. Lancaster, Some further notes on "Cascadeability", Cable Television Engineering, Vol 10, No. 7, December 1975

[5] S. Narayanan, Transistor distortion analysis using Volterra series representation, BSTJ, Vol XLVI, pp 991-1024, May-June 1967

[6] R.G. Meyer, M.J. Shensa, R. Eschenbach, Crossmodulation and intermodulation in amplifiers at high frequencies, IEEE J. Solid State Circuits, Vol SC-7, pp 16-23, February 1972

[7] A. Prochazka, R. Neumann, High frequency distortion analysis of a semiconductor diode for CATV applications, IEEE Trans. Consumer Electronics, Vol CE-21, No. 2, May 1975