FUNDAMENTAL RELATIONS IN CATV COSTS ENGINEERING

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Some of the fundamental relations between costs and technical parameters are derived in exact form. Others are shown in a general manner, graphically. Such analysis is useful in applying historically proven cost minimization techniques.

If a man buys 200 apples and 50 oranges at a total cost of 20 dollars, and then buys 200 oranges and 50 apples at a total cost of 42 dollars and 50 cents, what was the cost of each apple? Do you remember problems like this when you were in school? And were you tempted to answer "Who cares?".

That, of course, might be a reasonable attitude for a kid who wants to get out of the classroom and into a hockey game. But the problem can have some interesting aspects to a business man. Suppose the man in the problem is a storekeeper, and suppose further, that he has simplified his accounting system so that he enters the first purchase as:

250	Fruit	$20.00

On this basis he finds his unit cost to be 8 cents per fruit and decides that a markup to 15 cents will take care of his overhead costs and provide him with a nice little profit. Now, suppose that the oranges sell very well but the apples sell more slowly, so he now orders 200 oranges and 50 apples, and gets a bill for $42.50. He now enters:

250	Fruit	$42.50

and finds that each fruit now costs him 17 cents. Galluping Inflation! He has to raise his prices. His fruit sales drop. He still sells some oranges, but nobody buys apples.

Sometime after the bankruptcy proceedings were over he found that his real costs never actually changed. Apples were costing him 5 cents and oranges 20 cents.

Of course, no one would set up an accounting system where you averaged the costs of apples and orange in this manner. Or would they? Do you really know, for example, what it costs you to use 3/4" cable in your trunk rather than ½" cable? Or to provide 3 dB more signal to you customers? Or is the extra cost the same with a potential of 100 customers per mile as with 400 customers per mile?

As long as you are making a profit you may be able to take the schoolboy's attitude of "Who cares?". But unless you know how to find the answer to these and similar questions, you certainly cannot know whether you are minimizing costs, or for that matter, even taking steps which may lead to future losses.

The solution to the problem of our hypothetical fruit dealeris really very simple. He merely had to be sure that his accountant and bookkeeperknew, and kept track of the differences between apples and oranges. He would then have had the necessary information to make much better decisions about purchasing, pricing, etc.

In a field as technical as CATV, costs are affected by many technical parameters in many diverse ways and it is obvious that the decision making process must involve a considerable amount of technology. Some of the relations between technical parameters and costs are very simple. Others exhibit varying degrees of complication.

KELVIN'S LAW

In 1881, Lord Kelvin demonstrated that, in the case of electric power transmission, the most economical wire size is that where the annual interest on the investment in the wire is equal to the annual cost of the energy lost in the wire resistance.

This result, commonly known as Kelvin's Law, may be one of the first applications of costs engineering procedures.

Of course, stated in this way, Kelvin's Law is only exactly true if the cost of the wire is exactly proportional to the amount of metal in the wire. However, it can be shown that the principle involved can be stated in a slighly different form so as to be exactly true in those cases where partial costs vary in opposite ways with respect to a technically definable parameter.

First, separate those costs which vary with the parameter from those which do not.

(1) $C_t = C_f + C_v$ i.e. total costs equal the sum of fixed costs and variable costs. (relative to the parameter in question)

Taking the derivative, we obtain:

(2) $\frac{dC_t}{dP} = \frac{dC_v}{dP}$ Where P is the parameter involved. If there is a minimum cost, it will be possible to solve

(2a) $\frac{dC_v}{dP} = 0$. If not, the sign of the expression will indicate the direction in which P should be varied in order to reduce costs.

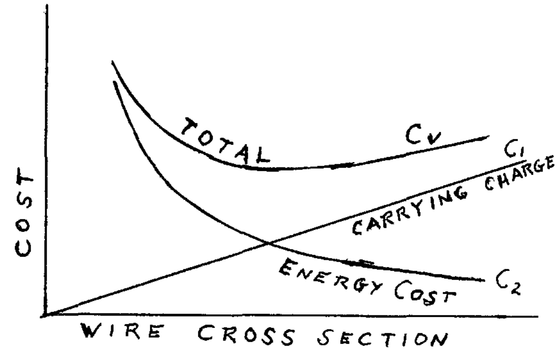
A very common situation is one in which the variable costs vary with the parameter through two (or more) mechanisms affecting cost in different ways. Let:

(3) $C_v = C_1 + C_2$ Then (2a) requires

(4) $\frac{dC_1}{dP} = -\frac{dC_2}{dP}$ which will determine the conditions of minimum cost. Note that the existence of a minimum requresthat C_1 and C_2 vary in opposite ways with P.

Fig. 1 shows the relations assumed by Kelvin in his derivation. Notice that the occurrence of the minimum at the point of equality of the two costs is dependant on the fact that one varies directly with the parameter (area of wire), and the other varies inversely to it. The rather broad minimum is also typical of many situations of this type. In this case, the broad minimum suggests that the additional investment which could provide either better voltage regulation or capacity for future growth would be rather small and ought to be investigated.

Although Kelvin's Law was originally applied to the problem of minimizing annual costs, the principles are applicable to the case where total costs vary in opposite ways with respect to any parameter. A rather simple and instructive case is that of a dead run trunk. In this case we have two costs which will vary with the attenuation of the cable which might be used. One, the total cost of amplifiers will increase in dirct proportion to the cable attenuation, and, two, the cost of the cable will decrease with the cable, although not in a simple inverse proportion. The situation is shown in Fig.2.



$$C_1 = K_1 A \qquad C_2 = K_2' I^2 R \simeq K_2/A$$
$$\frac{dC_1}{dA} = K_1 \qquad \frac{dC_2}{dA} = -\frac{K_2}{A^2}$$
$$A = \sqrt{K_2/K_1}$$
$$C_1 = C_2 = \sqrt{K_1 K_2}$$

Fig 1

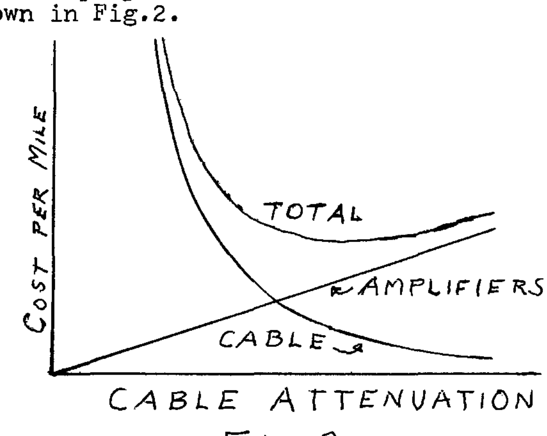


Fig 2.

The total variable cost in this case is:

(5) $C_v = C_c x l + C_a x n$

Where C_v= total variable cost, C_c= cable cost per unit length, C_a= cost per amplifier, l= length of trunk, and n= number of amplifiers. Equation (4) becomes:

(6) $$\frac{d(C_c l)}{d\alpha} = -\frac{d(C_a n)}{d\alpha}$$ and with

The obvious relation n = 1/G we get:

(7) $$\frac{d(C_a n)}{d\alpha} = C_a l/G$$ and

(6a) $$dC_c/d\alpha = -C_a/G$$

There are several ways to handle this result. The most obvious is to follow the method used in Fig.1 to derive Kelvin' Law. That is, find a simple relation between the attenuation, the cable dimensions and the cost. First, let us assume that the cost of the cable is directly proportional to the amount of material in them and that the different sizes are exact scale models of each other. In this case we find

(8) $$C_c=K_1 r^2 \qquad \alpha=K_2/r$$

since the "skin depth" will not vary appreciably with cable size. This leads to the relations $C_c= K_0/\alpha^2$ $K_0=K_1K_2$ and

(8a) $dC_c/d\alpha = -2K_0/\alpha^3$. Equation (6a) can now be solved for

(9) $\alpha^3=2K_0G/C_a$, leading to:

(10) $nC_a= 2lC_c$ or , the total amplifier cost should be just twice the total cable cost.

It is of course, unlikely that the cable cost will be exactly proportional to costs of the material in the cable. One very likely thing is that there will be some fixed costs involved in the cable. in this case we will have, instead of (8)

(11) $C_c=C_{fc}+C_{vc}$ with C_{vc} as in (8)

Now, instead of (10) we will have

(12) $nC_a= 2l(C_c-C_{fc})$, and the lowest cost for the trunk now requires that the total amplifier cost be lower than twice the cable cost. As a matter of fact it is possible and even likely, that the variable portion of the cable cost will not vary exactly as the square of its diameter but in some manner slightly different due possibly to manufacturing differences.

The preceeding approach is probably more suitable for a cable manufacturer to use in developing a cable most suitable for a market, rather than for a system operator or designer attempting to set up a set of parameters leading to the best system at the lowest cost. However, the above relations do indicate some limits which are useful in approximations.

The cable operator, or designer, is more apt to be presented with the cost of cable in the form of a price list such as:

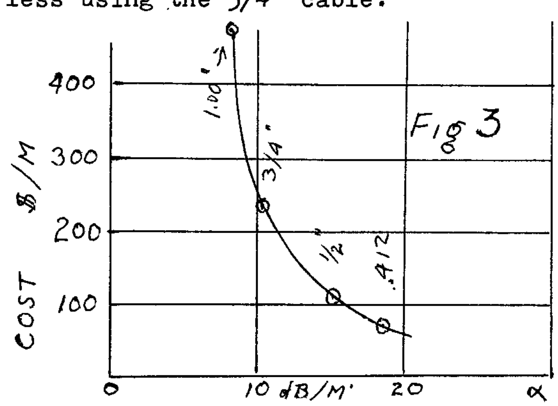
SIZE	ATTEN db/M	COST $/M
1.000	8.3	479.10
.750	10.5	238.50
.500	15.1	113.30
.412	18.4	78.15

Amplifier costs are available in a similar form, for example:

MODEL	REC. SPACING	COST
PDE	24 db	753.99
VDC	31 db	75.50

Suppose a designer has, for some reason selected the more expensive amplifier. He may still wish to select the cable size which will minimize investment cost. How should he go about it? One way is to design the system for each cable and then select the lowest cost design. Another way is to either analytically or graphically find a continuous expression for the variation of cable cost with attenuation so as to use equation (6a).

Fig. 3 shows a smooth curve joining points plotted from the price list, and Fig. 4 shows the negative of the slope of the curve in Fig. 4. Since the amplifiers cost 753.99/24 = 31.42 dollars per db, the designer will look for a cable for which the differential cost will be just $31.42 per db. He finds from Fig. 4 that this requires a cable with an attenuation of about 13.5 db/M. Since this cable is not for sale (except probably on special order) he still has the problem of selecting between 1/2" and 3/4" cable. He may decide that because 13.5 is nearer to 15.1 than it is to 10.5 and because 1/2" cable costs less per foot, he should select the half inch cable. And he will be wrong! His trunk costs would have been less using the 3/4" cable.



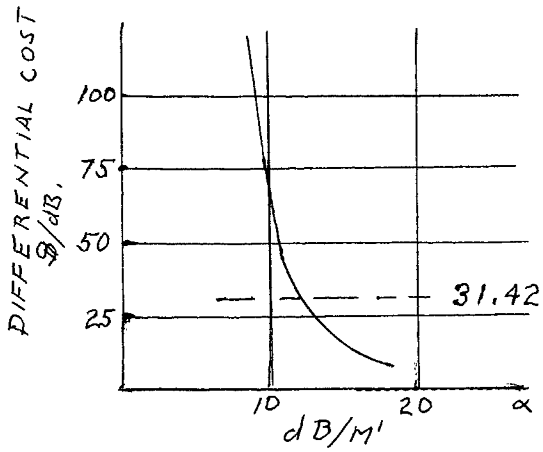


Fig 4

Of course, if cable of the optimum attenuation could have been obtained at the price predicted by the smooth curve, he could have done better than with either 3/4 or 1/2" cable. On the other hand it is important to see how to use the equations in a real world situation. The problem is easily solved if we look at Figs. 5 & 6. In Fig. 5, which is really only Fig.3 redrawn , we take into account the fact thatthere is no choice of cables between the values in our price list, and that we must go directly from one point to the next. Thus when we plot or differential cost ratios in Fig. 6 we obtain a step function as shown. The amplifier differential cost of $31.42 per db. is now seen to intersect the step function at the value for 3/4" cable.

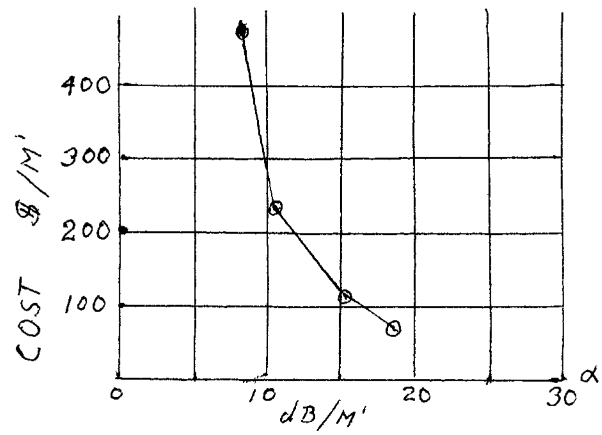


Fig 5

Fig. 6 makes it readily apparent that, given the cable attenuations and cost in our list, the minimum investment in cable and amplifiers will occur under the following conditions:

CABLE SIZE	AMPLIFIER COST $/db.
1.000	over 109.36
.750	27.22 to 109.36
.500	10.65 to 27.22
.412	under 10.65

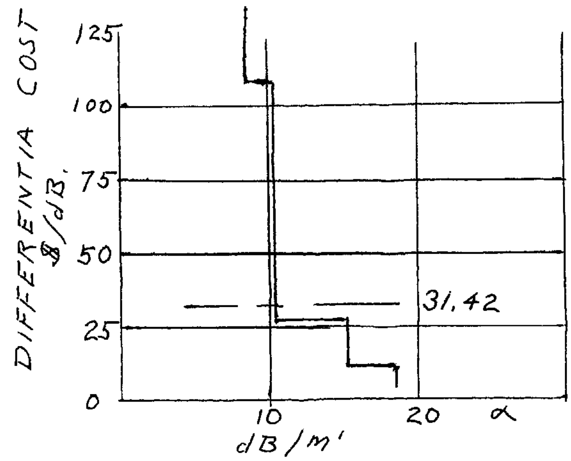


Fig 6

Thus, if one is considering a system small enough to allow the use of the very inexpensive amplifier with 31 db. gain at $75.50 shown in our price list, ($2.44/db) the least investment will occur with 0.412 or perhaps even cheaper cable. On the other hand if performance requirements indicate the probable necessity of the use of 1" cable, it would certainly be desirable to look into the possibility of obtainig amplifiers with high enough quality to justify a price of over $109.36 per db.

More exact results can, of course be obtained if installed and balanced costs are used instead of catalog prices, including any differental costs involved in connectors etc. On the other hand, corrections such as these will not change the results unless the situation is close to a transition value. Under these circumstances, it will however, usually be more productive to consider the effects of the decision on operating costs.

SIGNAL DISTRIBUTION

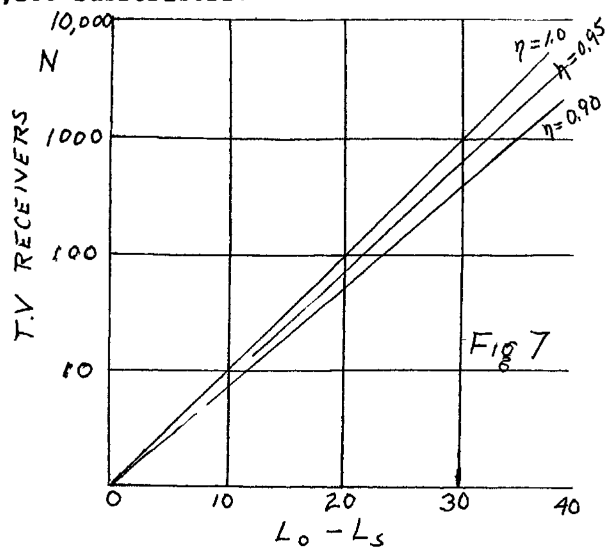
In the case of the dead run trunk, the problem required only the obvious relation between the number of amplifiers and the cable attenuation per unit length. In the distribution portions of the system there is the problem of cost per subscriber in addition to that of cost per mile of plant. These two cost measures can be related without too much difficulty by taking into account the subscrber density (actual or potential). Since some of the relations are a little intricate, it is probably worth while to examine a few idealized situationsbefore investigating the general problem.

The number of subscribers which could be fed from a single amplifier if we could somehow feed them with 100% efficiency is:

(13) $N=P_o/P_s$ where P_o is the amplifier power output and P_s is the required power input to the set. This can obviously be written inthe form:

(14) $10 \log N = L_o - L_s$

where $L_x = 10 \log P_x$. This relation is shown in Fig. 7. With an amplifier output level of 40 dbmv. and a required set level of 5 dbmv., one amplifier could serve 3,160 subscribers.



To approximate a real situation, the effect of splitting losses can be included in the equations. This is most readily done by assuming the use use of cascaded two-way splitters, each operating at an efficiency η . The number of sets which can be served is then:

(15) $$N=(P_o/P_s)^{\left(\frac{1}{1-\log_2 \eta}\right)}$$

Splitting losses are usually of the order of a few tenths of a db per split, corresponding toan efficiency of between 90 and 95%. The number of sets corresponding to splitting efficiencies of .90 and .95 are also plotted in Fig. 7. It is interesting that efficiencies of even 90% will only drop the number of sets in the previous example from 3160 to 1090. Splitting losses by themselves are obviously not the major limitation on the number of subscribers which can be fed from an individual amplifier.

1000 TV sets could be easily packed into a cube about 25 feet on a side, and the longest run of cable required to feed them from an amplifier in the center of the cube is less than 30 feet. It becomes obvious that the fundamental problems in distribution are not a great deal different from those in trunk. It is still largely a matter of getting the signal from one point to another through cable.

In the distribution portions of the system, however, there is the problem that the losses due to tapping do not follow the same mathematical law as the losses due to cable attenuation.

The attenuation of signal in the cable is exactly analogous to the increase (or decrease) of principal under the conditions of continuously compounded interest. (In the case of the cable there is a negative interest factor - the attenuation.) The tapping of fixed signal levels to the subscribers is likewise the exact analogy of fixed annuity payments at intervals corresponding to the subscriber density. In what follows, the analogy be with an annuity paid out continuously at a fixed rate, rather than in lump sums for simplicity, The extension to the case of discrete real tapping can be easily made if the particular situation should justify it.

First, define:-

$P(x)$= The power being propagated through the cable at point x.

P_t = The tap level required per subscriber

$a = -\frac{dP(x)}{dx} \cdot \frac{1}{P(x)}$ = attenuation per unit length of the cable.

D= The subscriber density in subscribers per unit length.

$A=DP_t$ = Thesignal power tapped off per unit length for subscribers.

The equation representing distribution conditions is:

(16) $dP/dx = - aP - A$ This is readily solved to obtain:

(17) $P=P_o e^{-ax}\left(1-\frac{A}{P_o a}\left(e^{ax}-1\right)\right)$ where

P_o is the amplifier output. If A varies along the span, equation (17) must be written:

(17a) $P=P_o e^{-ax}\left(1-\frac{1}{P_o}\int_0^x A(x)\, e^{ax} dx\right)$

With subscribers tapped off at fixed locations rather than continuously as is assumed in (17), this equation will still hold if the integral is taken in the

Lebesgue- Stieltjes sense. Note that P_o needs merely to be the signal into the section of cable under consideration, as for example, a section of cable after a split.

Equation (17) can be rewritten as:

$$\ln \frac{P}{P_o} = -ax + \ln\left(1 - \frac{A}{aP_o}(e^{ax}-1)\right) \quad (18)$$

If we set $P=P_i$, the power level into the next extender (or into the line termination) the left hand side of the equation becomes the amplifier gain, (or actually, the total span loss), and it is possible to solve for the max. spacing.

$$l_1 = \frac{G}{A}\left(1 + \frac{1}{G}\ln\left(\frac{a+A/P_o}{a+A/P_i}\right)\right). \quad (19)$$

This result could be plugged into equation (5), using the relation $n=l/l_1$ for the number of amlifiers in a given distance l, to obtain the lowest investment cost for cascaded line extender situations. On the other hand, this equation is applicable to a wide range of situations, including feeders which split, etc. It is therefore probably more worthwhile to examine (19) in some detail to see more of the information it contains. A few points are immediately apparent;

1. The expression would be the same as for an untapped line, $l_1=G/a$, except for the expression in parentheses which represents a spacing factor due to subscriber tapping.

2. Since $A=DP_t$, it is obvious that doubling the subscriber density will shorten spans by the same amount as increasing tap levels by 3 db. etc. and vice versa.

3. As subscriber density is reduced, the spacing factor approaches unity, which is the same condition as in a clean trunk.

4. As subscriber density increases, the spacing factor approaches zero and we approach the situation of very dense packing considered in equation (13) where the cable loss becomes less important.

Equation (19) points out that the spacing (or feeder lengths) in distribution areas depends on subscriber density in a very non-linear way. A somewhat clearer understanding of this dependance can be achieved by writing the spacing factor in a slightly different form.

First, it is desirable to introduce a reference density, the saturation density. This is the density at which the loss of signal per unit length in the cable due to tapping is equal to the loss dur to cable attenuation. This density, in the case of 100% efficient tapping is:

$$D_s = \frac{aP}{P_t} \quad (20)$$

Fig, 8 shows how the saturation density varies with signal level in the cable from a low level equal to the tap level up to a level 35 db higher than tap level, such as might occur right at an amplifier output.

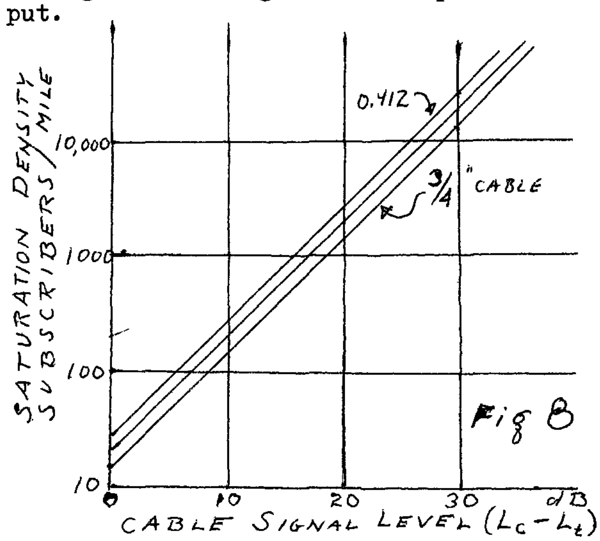


Fig 8

At the highest cable level shown, the saturation density of approximately 50,000 subscribers per mile corresponds to about 1000 in 100 feet, which is very close to the densely packed case. On the other hand, near amplifier input levels, or line terminations, the saturation density can be in the order of tens of subscribers per mile. In a real system, we can expect to encounter actual densities ranging from close to saturation density to as low as one one-thousandth of saturation density.

It is possible to write the portion of equation (19) representing the spacing factor as:

$$s = \left(1 + \frac{1}{G}\ln\left(\frac{1+D/D_{so}}{1+gD/D_{so}}\right)\right) \quad (21)$$

where $g = P_o/P_i = e^G$ and D_{so} is the saturation density at the amplifier output. Figs. 9 and 10 show how the spacing facto varies with G and D/D_{so}.

It can be seen that except for very low densities, an increase in gain (spacing in db) begins to decrease the spacing factor noticeably at even moderate values of gain. Design phiosophies which attempt to reduce costs by the use of high gain amplifiers could under the right circumstances easily run into the law of diminishing returns.

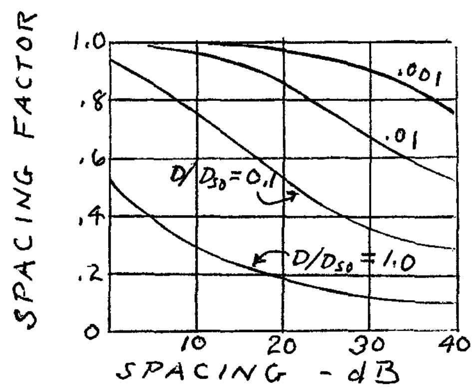


Fig 9

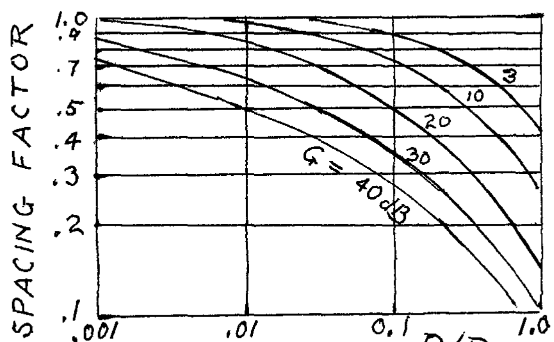


Fig 10.

The fact that taps are not 100% efficient devices, but involve some losses of their own can be included in the calculations in the following manner.

In order to take into account the fact that the taps may handle the through signal with a different efficiency than they handle the tapped off signal, let:-

η_1= the efficiency with which the through signal is handled.

η_2= the efficiency with which the tapped signal is handled.

The effect of η_2 requires that an amount of signal equal to DP_t/η_2 be tapped off rather than just DP_t. This is just the same as if their were an effective density of $D_e=D/\eta_2$. With average tap efficincies this would represent an increase in density of approximately 10% and is in many cases less than the effect of unused tap ports, and the effect of steps in tap values available.

The effect of η_1 will be to reduce the cable signal level by an amount $(1-\eta_1)P$ for each tap installed. Since the number of taps per unit length will be equal to (or greater than) D/N_t, where N_t is the number of output ports per tap, the signal will be reduced by an amount equal to $D(1-\eta_1)P/N_t$ per unit length. This is the same as if the cable attenuation were increased by an amount, $D(1-\eta_1)/N_t$. At low densities, this will generally have little effect on costs. However, at higher densities the effect can be noticeable.

ANNUAL COSTS

The same principles can obviously be used in the minimization of annual costs as have been shown to apply for investment.

Fig. 11 shows a first degree approximation of a method by which the selection of cable size might be modified so as to mimize annual costs. The basic relations shown are those of initial cost from Fig. 2, but with the scale of the vertical axis changed in a proportion to reflect total annual cost on the investment. To this another curve is added to reflect the items of annual operating cost affected by the choice of cable.

The first and most obvious of these costs to take into account are those which are directly proportional to the number of amplifiers in the system, amplifier maintainance and energy costs. With this relation added to the chart, it is immediately apparent a higher initial investment in cable can reduce total annual costs.

The next step requires a more thorough investigation of the effects of amplifier gain, operating levels, system size, and general design philosophy on operating costs. The analysis so far indicates that the use of higher gain amplifiers, running at higher levels will frequently allow the the use of lower cost cable in a way which might reduce the investment in both amplifiers and cable. It might be expected that the reduction in number of amplifiers occasioned by this approach would also reduce the maintainance and energy costs, ending up with an extremely low investment and an almost maintainance free system. That there is clearly a limit to this process, can be shown by the calculation of the requirements for a single amplifier to operate a large or even moderately sized system. The energy costs alone become astromical. On the other hand there are limits which begin to show up in the real world.

It has been proven by several methods, that if <u>any</u> limits are set on the allowable degradation of signal quality due to amplifier distortion and/or noise buildup in the system, there will be an optimum amplifier gain which will the construction of either the largest system which can provide the signal quality or a smaller system with the maximum operating safety margin without exceeding the limits. This optimum gain will be between 4.3 and 13.0 db depending on the nature of the limits . Als associated with the gain will be an optimum signal level for operation. Increasing the deviations from these optimal conditions will reduce the operating safety margin first and eventually cause the originally established limits to be exceeded. All present systems are operated at gains and levels higher than the optimal values in order to keep down the first costs.

It is very seldom proposed that a system be designed so as to just meet quality requirements with no operating margin because of the effects of operating safety margin on operating costs. It is somewhat difficult to set up exact mathematical relations in this area. Individual variations in maintainance methods, procedures and efficiency often make exact base data difficult to obtain. However, it is possible to show some of the important general relations by graphical means, and point up possible danger areas.

Limit specifications can be approachin several ways, thereby reducing operating margins. Amplifier levels can be increased or decreased or gain can be increased, thereby both raising output and lowering input levels; lower cost cable (with higher attenuation, or other reduction in performance) can be used with more amplifiers, or higher gain amplifiers, or any combination of these situations.

Fig. 12 shows two possible limit situations. One way in which the operating margin might be reduced so as to come out exactly at the specification limits is to use a higher loss cable. The point marked A in the figure represents such a possible situation. If cable of any higher attenuation were to be used, it would be impossible under any conditions of maintainance to meet the requirements. Even at the exact limiting value, it would be necessary to maintain all system levels, responses etc. at <u>exactly</u> the design conditions if limits on picture quality are not to be exceeded. In the real world, this is impossible at any realistic cost.

If now, lower loss cable is used, the maintainance costs will come down as as the increased safety margin makes it easier to keep the system within limits. The exact rate at which the costs will decrease will depend on a great many factors but will certainly be quite rapid at first and eventually approach the straight line straight line curve of Fig. 11.

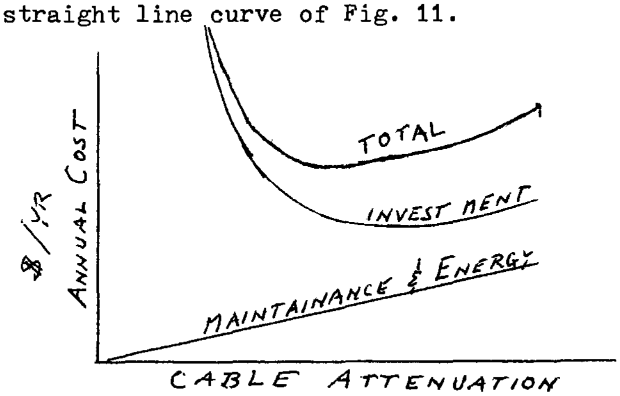


FIG 11.

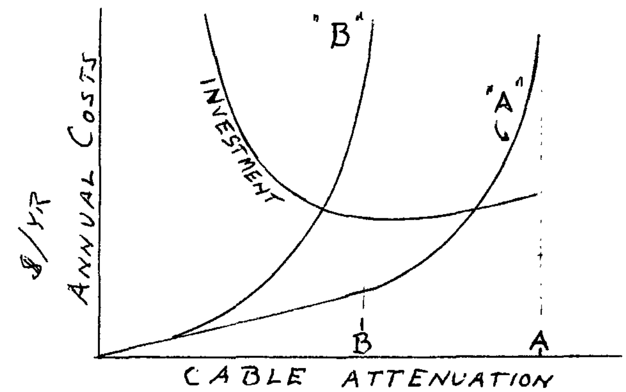


Fig 12

In the case represented by the curve "A" of Fig 12, the operating margin will normally be large, even if the design were based on first cost alone, and for all practical purposes, the situation reduces to the approximation used in Fig. 11. However, if a larger system is under consideration, or if a different design philosphy is adopted, the limit conditions, and therefore the maintainance cost curve can move towards the left of the figure as shown by curve "B". This situation obviously requires either an increase in investment costs or a different design philosophy if total annual costs are to be minimized.

CONCLUSION

The relations developed here are not complete by any means.The intent has been to show how much more remains to be done. However, it has been shown that in a great many areas, exact expressions can be written relating costs and importamt technical parameters.

For reasons of simplicity, most of the relations have been developed as continuous functions. Obviously, much of the hardware represented by the parameters is not available in continuous form.

However, just as was done in the case of cable values, which were available only in steps, it is usually possible to interpret the results in terms of step or impulse functions.

In the area of maintainance costs much more work is obvioulу needed. The relations between equipment costs, system costs, and maintainance costs contain many complex factors. In addition, much basic maintainance cost data has not been assembled in as simple and exact a form as is the case with hardware and design costs.

If there is any moral to this story, it would appear to be best expressed as an answer to the frequently made comment that "You can't compare apples and oranges".

The most reasonable answer appears to be "Then we had better not get into any enterprise as highly technical as the retail fruit business."