"THE DESIGN OF EXPANSION LOOPS FOR REDUCING FATIGUE IN COAXIAL CABLE INSTALLATIONS"

B. D. Garrett

Superior Continental Corporation R & D
Hickory, N. C.

ABSTRACT

The mechanisms of fatigue in non-ferrous metals and designing against them in coaxial cable is a complex subject and one which can be especially difficult when designing for a 25-year plant. The life of any one expansion loop is determined by a plurality of variables, such as environment, installation temperature, span length, cable size and expansion loop position and dimensions, all of which are discussed. The use of brittle coatings for full-field strain analysis, strain gage measurement, and extensive fatigue testing have been conducted to better establish the mechanics of various expansion loop designs and to evaluate their potential worth. This data has then been transformed into analytical approximations for the engineer in order that loops of better known life can best be designed to the particular system.

INTRODUCTION

The existence of a differential thermal expansion between coaxial cable and associated steel support wire in aerial cable plants is a subject well known by CATV systems engineers for many years. Its presence has been manifested in a number of undesirable forms such as cable snaking, connected pull-out or loosening, and outer conductor and lashing wire failure. To circumvent as many of these problems as possible the industry has formed two design philosophies; one utilizing the addition of expansion of various sizes and shapes to accommodate the changes in cable length, the other to install no expansion loops and to lash doubly tight to restrain the cable expansions and contractions.

The successes of both approaches to date have shown to be only limited in number and for those systems which have survived the first few years, only extended years will demonstrate their virtues. For the better part, a proportionately large number of systems have experienced fatigue failures and for this purpose the following project was initiated in June of 1972. Its objective was to investigate thermal fatigue failures in coaxial cable and to make recommendations to reduce their occurrence toward a plant design life of 20 - 25 years.

INITIAL INVESTIGATION

In order to begin the investigation, resolution of the preferred design philosophy was of first consideration. The concept of no expansion loop is one which upon first observation was the more desirable. For the argument of simplicity the thermal expansion of the coaxial cable was assumed to be that of aluminum, $\alpha_a = 13.3 \times 10^{-6}$ in/in °F. This would discount the influence of the center conductor through the dielectric which would certainly be negligible in consideration of the low pull-out strengths and use of copper clad aluminum center conductor in many cables of .500 inches and larger. The thermal expansion of the support wire was established as that of steel, $\alpha_s = 6.0 \times 10^{-6}$ in/in °F, which results in a thermal differential coefficient, α_d, of $\alpha_d = \alpha_a - \alpha_s = 7.3 \times 10^{-6}$ in/in °F. Under the assumption that the cable and support wire were perfectly lashed together, the resulting stresses, σ, within the cable would be approximately $\sigma = E\varepsilon = E\alpha_d \Delta t$ where E is modulus of elasticity of aluminum, ε, the strain of the aluminum and Δt the change in temperature. The strain term, ε, represents a unit elongation, or contraction, $\Delta l/l$ and is more commonly expressed in terms micro strain, $\mu\varepsilon$, representing 10^{-6} in/in, ft/ft etc. From this it could be seen that the resulting strains and stresses in a no loop design are relatively low compared to that found within expansion loops. For this reason, along with installation ease, the no loop concept appears desirable.

However, further considerations demonstrated the opposite to be true. Unlike ferrous metals, aluminum does not exhibit an endurance limit, that is, there exists no stress below which a fatigue failure will not occur. The resistance to fatigue in aluminum is therefore given by a fatigue strength figure, in psi, which corresponds to the stress range at 10^8 or 5×10^8 cycles to failure. By staying within the elastic or lower elastic limits of the material, it does increase the expansion loop life, but for this reason does not insure that the expansion loop will not fracture as may have been assumed. By lashing the support wire and cable into a integral unit, it becomes a load bearing unit of the span to experience not only all stresses resulting from daily and seasonal temperature changes, but also span vibration and galloping, Because of the unknown cycles associated with the latter, much uncertainty lies in estimating cable life in light of the absence of an endurance limit. This uncertainty coupled with knowledge that no system can be perfectly lashed, and that large accumulated displacements will be transmitted to small areas not designed to accept large displacements, such as unlashed areas beside a pole or at equipment, plus the existence excessively large forces increasing with the square of cable diameter acting upon connectors made the approach undesirable.

The utilization of expansion loops however, recognizes the existence of changes in cable length with changing temperature and is designed to accommodate them. Its purpose is to efficiently "absorb" the strain energy of the expanding cable span and to evenly distribute it over its own length. By doing so cable stresses can be minimized, connector loading significantly diminished and system life extended.

It is for this reason that all further investigations were related to expansion loops. The validity of the expansion loop concept does presuppose a properly designed expansion loop and it is for this purpose much of the work was conducted.

In the sequence of investigation, the first tests conducted were fatigue life testing. These tests were conducted to establish the initial elementary relationships between loop size and design, and life. They consisted of taking an expansion loop of known size and extending and contracting them given distances in repeated cycles until failure occurred. This was accomplished through the use of the apparatus shown in Figure I. The relatively simple device

FIGURE 1

consisted of a zinc diameter pneumatic cylinder, counter, limit switches, and cable clamps. For a condition of 1 inch excursion the sample was placed into the apparatus at its free length, clamped to the moving end at the left, and stationary clamp at the right. The limit switches were placed .50 inches to either side of free or neutral position. The test was then turned on and permitted to cycle until failure.

The fatigue life of any expansion loop, or any structure for that matter, is a function of numerous variables. Many of these variables are material properties such as ultimate tensile strength fatigue strength and surface roughness, physical dimensions such as cable diameter, wall thickness, and stress profile or environment. In an attempt to isolate as many variables as possible all tests were conducted with cable taken from the same reel except, of course, in those tests specifically conducted to evaluate the influence of material or cable design variables. All tests were also conducted about the free or neutral length of cable. This means that for any excursion half of that excursion was toward loop extension and half toward loop compression. The influence of a stress bias or mean stress has a considerable influence in extending or reducing the fatigue life of an element. For this reason, the conditions of zero mean stress was selected.

By taking each particular expansion of interest and cycling it to failure at each of several excursions, curves somewhat similar to the conventional S-N curve could be generated for each design. Some of this data is shown in Figure 2. In the early work, the curves for the round bottom loop or drip loop, and shallow extended expansion loon were the first to be run, and their relative fatigue lives are much shorter than the remainder of those on the figure. Once the general nature of the expansion loops were established, the improved loops were developed based upon qualitative and analytical data.

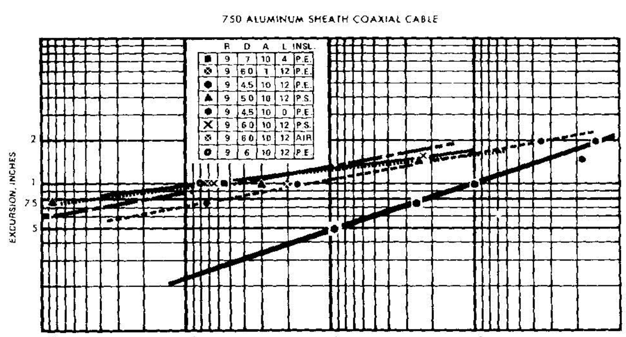

FIGURE 2

The data presented in Figure 2 is an excellent illustration of the relative performances of various design configurations in terms of low cycle and higher cycle characteristics, but says nothing about stress distributions, bending moments, or force requirements to displace the loop, all of which are important to the placement of connectors; nor does it say to assume that one cycle per day of excursion amplitude equivalent to the variation in mean annual daily high and mean annual daily low would be a gross over assumption. To determine the strain distributions, force requirements, and estimated life will require an understanding of its mechanics.

BRITTLE COATING ANALYSIS

In the preliminary mechanics study, brittle coating were utilized to examine the full-field strain distributions. Brittle coatings are nothing more than lacquer of controlled sensitivity which are sprayed on the particular surface of interest. Unlike a strain gage, the brittle coating is capable of providing strain distributions over a complex surface as opposed to a single point. When the structure is loaded, or expansion loop compressed, the tensile strains on the surface of the outer conductor cause the coating to crack when the strain level has reached its calibrated sensitivity level. The formed cracks are normal to the principal tensile strain direction and the area covered by the cracks maps the area at or above the calibrated strain level. By incrementally increasing the loads and tracing the cracked areas at the end of each increment it is possible to obtain qualitative analysis of the stress distributions within the expansion loop. The loops examined by the brittle coating techniques were classified into three general groups illustrated in Figure 3. They are the extended expansion loop, drip loop, and 360° expansion loop.

GENERAL SHAPES

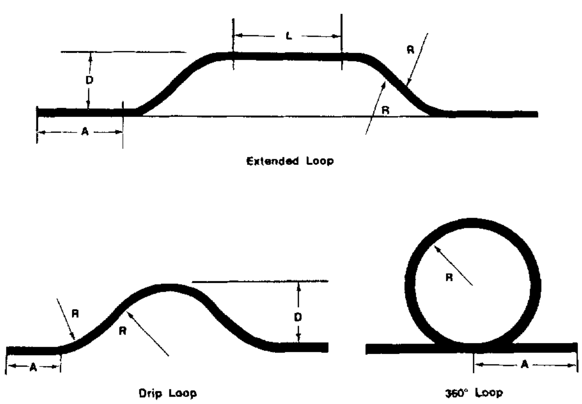

FIGURE 3

The drip loop type expansion loop illustrated is probably the most commonly used expansion loop. The extended expansion is also presently used in some systems and the 360° expansion loop probably not at all. The results of the brittle coating examination on these three basic designs are shown in Figure 4. Although the strains are not shown in exact proportion, some quantitative data was derived from the testing and a number of interesting points were found and are illustrated in Figure 4. One of the most important of these points is the existence of what has been called the nodal point. They are signigicant because - 1. they represent a point of zero stress, and 2. the vertical distance measured from a horizontal line passing through them to either the center line of cable or bottom of the loop regulates, for the better, the bending moments and stress at that point.

STRAIN DISTRIBUTIONS

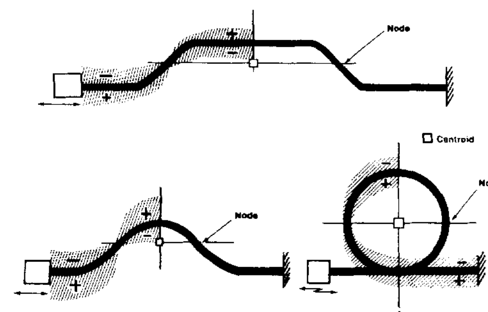

FIGURE 4

The location of the nodal point for symmetrical expansion loops of the three general forms mentioned is found by passing a line (horizontal in this case) through the centroid of the expansion loop. The technique for location of the centroid will be discussed later. The actual existence of this point is not academic and represents the point at which the stress switches from tensile to compressive or compressive to tensile. Verification of this point was found by accident in tests designed to evaluate the significance of cable dielectric to fatigue life. In this particular test, an expansion loop was formed from cable with no center conductor or dielectric. In the formation of the loop, a deep buckle was formed in the cable by accident at the nodal point. Had this same buckle been at a point of maximum stress, mid-position on the flat length of an extended expansion loop, it would have never survived 100 cycles. As it was, the loop was cycled to its anticipated life of approximately 7300 cycles and the failure did not occur at the buckle but rather at the point of maximum stress, the mid-point of the straight section.

Locating the points of maximum stress is important not only in terms of locating the point of failure, but also in terms of selecting the point to or not to place connectors. Connectors for aluminum have historically been trouble spots, not only in the CATV industry but in the telephone and power industries. The inherent properties of aluminum, specifically high coefficient of expansion, relatively inert oxide, and high creep and relaxation rates make it most difficult to connect to much less aggravated by large bending moments and thrust loading. Refering to Figure 4, the maximum bending and stress points are located at the top of the radius for the drip loop and

360° loop, and at mid-point of the straight section, L, on the extended expansion loop. It is at these points that connectors should never be attached. By placing a connector in the center of the drip loop type expansion loop, a situation very similar to bends created at equipment results. Any time the cable deviates from its standard centerline, bending moments and high stresses will result if the cable is subjected to thrust loading from thermal expansion. The magnitude of these stresses will vary with the thermal strain of the cable and the distance in which the cable deviates from the centerline to the equipment closure connector. If this distance becomes excessive, it should be isolated with the use of expansion loops at either side or at least one side and tightly lashed or clamped between the expansion loop and the equipment closure. The next highest stress location is found at the expansion loop ends. These stresses are also the result of an applied bending moment and vary with vertical distance measured to the centroid of the expansion loop. So long as these stresses are less than those at the maximum stress point, they are of little consequence. If it is necessary to apply a connector in this area, the connector can be isolated from the bending moment by installing a two cable spacers and straps inline between the connector and the expansion loop end.

The 360° expansion loop, although somewhat of an academic novelty, was useful in demonstrating several important factors about expansion loops. The first is that the expansion loop performance does not necessarily vary with expansion loop size nor does expansion loop optimization relate with the design of a universal expansion loop which will operate at maximum utilization (minimizing the required number per system) in all systems. Such a loop cannot exist. Optimization should refer to designing the highest performance, longest lived, out of the smallest possible loop. The 360° loop is a highly optimized loop, but as mentioned earlier represents, for the better part, an academic exercise in that the installation of the loop is not possible in large diameter cable where there is not access to a free end. In those locations where an end is accessible, the depth requirement of loop would often make it undesirable in terms of maintaining clearance with existing aerial telephone plant. To illustrate the improvement between the efficiently designed and less efficiently designed expansion loop, the 360° loop and drip loop type expansion loop of equal radii were compared in brittle coating analysis. The drip loop design was first compressed .125 inches and the entire length from node to node cracked representing greater than 500 μ in/in strain. Since the entire area cracked with the first small excursion, no quantitative assessment could be made of the point of maximum strain at the bottom of the loop. The mechanics of loop would predict that it was much much higher. The same testing was conducted on the 360° loop but the initial compression was set at .250 inches, twice that of the earlier. The resulting strain field greater than 500 $\mu\varepsilon$ was a narrow strip approximately 1 inch long located at the bottom of the loop. It was not until a compressive displacement of .625 inches that the 500 $\mu\varepsilon$ strain field approached the nodal points of the loop. The same loop was the best performing loop in life cycle testing where the test was terminated after reaching well beyond the 100,000 cycle range.

All of the data generated for the particular classes of expansion loops was conducted upon what may be referred to as a laboratory model in that they were properly formed. The lives obtained from these loops would also represent the maximum possible in that they were properly formed. From those failures that returned from the field it was obvious that much of the problem was associated with proper formation of the loop. This is especially true of those loops that failed in less than one year. The drip loop type expansion loop which was the poorest performing loop of all those tested now became much much worse with improper installation. For this particular loop, the problem of proper installation is aggravated by having the point of maximum strain in the middle of a radius which is hard to properly form in large diameter cable with or without the use of a tool. Those loops of the drip loop design which returned from the field were often of what was referred to as a "jerk" loop - indicative of the installation procedure only. Because of the relative rigidity of large diameter cable and unavailability of properly designed tools, it was much easier to form the drip loop type expansion loop by pulling downward on the cable or pushing upward in two points over a wooden form. The resulting shape of the loop was much closer to a "V", characterized by three tight radii with straight sections between them. The strain distribution of the loop as represented in Figure 4 would then become completely distorted. Those strains at or adjacent to the nodal points would be very small but rise rapidly to the center of the radius at the bottom of the loop. The strains at this point would be highly concentrated and would extend well into the plastic regions of the aluminum. In this condition, the loop is capable of only a few cycles.

The approach of removing the point of maximum strain from a radius is an important design criteria. By doing so, the life of the loop is not critically dependent upon the skills of the particular installer. In the extended expansion loop, this is an important design benefit besides its low profile and improved life.

STRAIN GAGE ANALYSIS

Once the full-field strain distribution had been determined the next approach was to further refine the analysis with the use of strain gages. Although the strain gage does not provide a full-field strain distribution, once the point of interest has been located, the point of maximum strain in this case, the gage can be applied to that point to obtain a highly accurate quantitative evaluation of strains at that point.

The data derived from this particular testing is of considerable usefulness, not only for supplying much of the required data necessary to make approximate life calculations, but as an evaluation of the aluminum used in the outer conductor of the cable. By placing a strain gage at the point of maximum strain and measuring the strain versus varying excursions the data presented in Figure 2 can be readily converted to the conventional S-N curve for fatigue in bending. It can also be used for a very quick A-B comparison of various loop designs. Taking any two, or more, loops and placing the strain gage at the points of maximum strain and compressing or extending the loops to the same given displacement, the loop with the lowest strain will be the longer lived loop at that particular displacment. This procedure should be carried out at a number of varying displacements extending to the point of maximum design displacement to insure that the relative magnitudes of strain between the loops in comparison are true over the entire range.

This same technique was used to evaluate the significance of many design variables. Most of these variations could be justified analytically, but their true significance can be accurately measured with the strain gage and recorded on a strip chart recorder. In general, it was shown that increasing any of the dimensions D, L, R, and A (refer to Figure 3) would result in a decrease in strain as measured at the point of maximum strain and consequently increase loop life. The increase of dimensions R and A is of extreme interest because their increased length does not require additional cable in the formation of the loop and does not affect the profile dimension D. The influence of A and L on life specifically can be seen in Figure 2. The condition of L=0 is illustrated by the drip loop type expansion loop and for an expansion loop of dimensions R=9, D=6, and L=12 the number of cycles to failure are shown for A=1 and A=10. The life is much longer for A=10. The significance of A and L can also be calculated and will be demonstrated later. Increasing dimensions A and L also contributes to expansion loop life in another manner not readily calculated, and is in terms of the loops ability to accommodate excessively large displacements associated with environmental extreme conditions. By increasing dimension A, measured from the first cable strap to the point of tangency of the cable leaving center line, this part of the loop becomes an active contributor to the overall strain energy "absorption" of the loop plus provide an additional mode of loop distortion. This can best be illustrated in Figure 5.

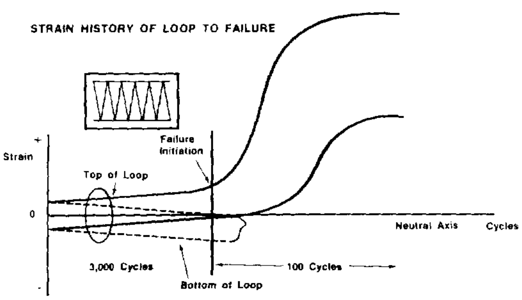

FIGURE 5

In Figure 5, a test was conducted to observe the entire life operation of an expansion loop through the point of failure. The test was performed on an extended type expansion loop with two strain gages mounted at the points of maximum strain. The lower channel, outlined by dotted lines, was mounted on the side that would normally be closest to the support wire. The second gage was placed on what would normally be the bottom side of the loop. The two lines given for channel is used to outline the limits of strain range measured peak to peak. On the actual recording, inside these limits are the strain cycles as depicted in the figure insert. For sake of illustration, the first 3,000 cycles have been highly compressed and the cycles beyond the initial point of failure expanded. However, the relative amplitudes of the strains within the illustration are maintained.

The most significant data derived from the region from 0 to 3000 cycle is the observance of the drift in strain range of the two channels. The occurrence of this phenomenon is the result of the loop dimension being distorted by the large excursion and the loop is seeking a new equilibrium shape and stress distribution. In the initial cycles this drift occurs at a high rate and continues to reach an equilibrium condition asymptotically. The particular test illustrates a condition where the loop failed prior fully approaching its equilibrium condition. This is normally the case in high displacement. For low levels of displacement, the observed drift occurs rapidly, meets its equilibrium conditions and continues to cycle at that level.

The ability of a loop to favorably distort when subjected to an excessively high excursion, or the likelihood of the strain ranges to drift is, for the better part, dictated by dimension A. The influence of A can be calculated for lower displacement where there is little distortion of the loop geometry, but is not so easily done for large displacement. Therefore, it can be said that the increase of dimension does contribute to loop life for low amplitude displacements but its greatest contribution is at high amplitudes of displacement. In tests conducted on the drip loop type expansion loop with a minimal A dimension, there was little evidence of drifting in the strain range and the stresses resulting from high displacements where heavily concentrated a point of maximum strain.

Once failure occurs, as indicated in Figure 5, the strains on the side of failure approach zero, and the entire geometry, and mechanics, of the system is altered. With the fracture on one side, almost always the side closest to the strand, the neutral axis of the cable is shifted, and the mean strain and strain amplitude of the remaining side increase rapidly until failure occurs. The stresses in this area reach far into plastic regions of the material and failure is more the result of gross deformation.

ANALYTICAL EVALUATION

Often, the engineer in the design of a system may envision what in his mind appears to be an improved expansion loop which better meets the requirements of the particular system presently in design, but because of possibly the unavailability of strain measurement equipment, and test fixtures, or the unavailability of time required to perform life testing, an analytical means is required to evaluate the concept. Once this relatively short operation is performed, the engineer can decide in a first order approximation whether the proposed technique is better, worse or further investigation is justifiable.

In order to perform such an analytical solution, a good understanding of the fundamentals of applied mechanics is necessary and specifically familiarity with the use of Castigliano's theorem. As it often is, people familiar with applied mechanics are not available and those that have been familiar with it have long since forgotten it from the lack of application. As an alternative, S. W. Spielvogel[1] has developed techniques for determining thermal stress in pipes in a simplified method based upon Castigliano's theorem.

As an example calculation, the stress distribution for the extended expansion loop of dimensions A=10, R=9, L=12, and D=6 in Figure 6 will be calculated. For brevity, the derivation of the general equations will not be made and it will be left to the reader to obtain verification from reference (1) if desired. The use of this technique assumes that the material remains within the elastic limits of the aluminum outer conductor and it is for this reason that the technique is applicable basically for comparative purposes. The stresses within even well designed expansion loops can reach plastic proportions but this is only upon large displacement and their calculation is of little consequence since this technique will only be valid at small displacements.

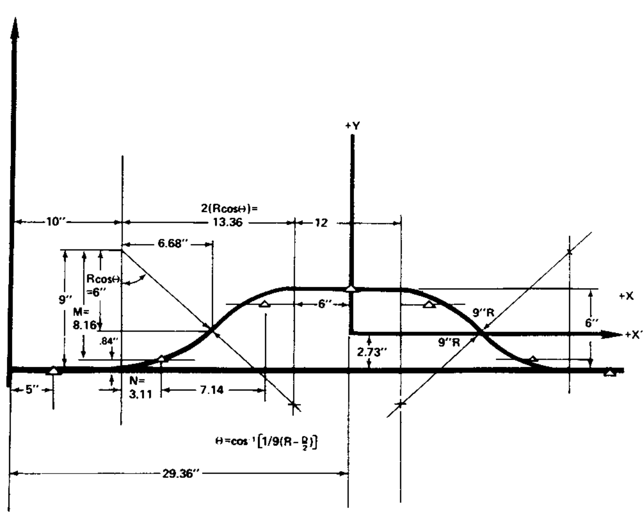

Figure 6

[1]S.W. Spielvogel, Piping Stress Calculation Simplified. New York: Byrne Associates, Inc., 1955, fifth edition, pp. 1-21.

In order to begin the sample calculation, some initial information must first be obtained. The first is the dimensions of the cable to be used and for this example a .750 inch diameter cable of wall thickness, $t = .033$ inches was used. From this, the moment of inertia, I, of the cable can be calculated from the equation,

$$I = (d_o^4 - d_i^4)/64 \tag{1}$$

where d_o = outside diameter and d_i = inside diameter.

$$I = \pi(.750^4 - .684^4)64 = .00478 \text{ in}^4$$

The next step required determining the thermal expansion, or contraction, of the cable length capable of expanding into the loop. This was calculated under the conditions of a 50°F temperature increase for an effective cable length of 115 feet. The resulting change in length, Δx, was then

$$\Delta x = l\alpha_d \, \Delta T \tag{2}$$

where l = effective cable length, α_d = differential coefficient of expansion, and ΔT = change in temperature.

$$\Delta x = (115)(7.3\text{x}10^{-6})(50) \text{ x } 12 = .5 \text{ inches}$$

The last remaining data is the modulus of elasticity, E, for aluminum which is 10 x 10^6 psi.

The next step in the procedure is the calculation of the centroid of the expansion loop. The best method for doing this is by constructing a sketch of the loop as in Figure 6, locating a convenient coordinate system X',Y', and dividing the loop length into straight and curved segments as illustrated. The calculation of the location of the loop centroid is performed simply by multiplying the length of the straight cable segment times the distances from the cable segment centroid, midpoint, to the origin of X', Y'. In order to do this for curved section a modified length, l, and arc centroid must be found. Taking arc CD as an example, the modified length, l, is computed for an arc radius, R, of 9 inches.

$$K = \frac{1.65\,(d_o - t)^2}{4tR} = 2 \tag{3}$$

The modified length, then becomes,

$$l = R\theta \text{ x } K = 15.12 \text{ inches}$$

where θ is the angle θ expressed in radians.

The calculation of the centroid is somewhat more involved, but again simple. In order to do this, again for arc CD, the distance from the arc center to centroid, c, is found from the equation,

$$c = \frac{R \sin 1/2\,(\theta_2 - \theta_1)}{1/2\,(\theta_2 - \theta_1)} = \frac{R \sin 1/2\,(\theta)}{1/2\,(\theta)} = 8.76\text{in.} \tag{4}$$

Knowing c, the coordinates (m,n) of centroid from a coordinate system whose origin is located at the center of the arc can be calculated from equations 5 and 6,

$$m = c \sin 1/2\,(\theta_2+\theta_1) \\ = 8.76 \sin 1/2\,(90 + 48.2) \\ m = 8.16 \text{ inches} \quad (5)$$

$$n = c \cos 1/2\,(\theta_2+\theta_1) \\ = 8.76 \cos (69.1) \\ n = 3.11 \text{ inches} \quad (6)$$

Where θ_2 is the angle measured from the horizontal to line segment OC and θ_2 is the angle measured to line segment OD if O is the center of arc CD.

The centroid can now be calculated in a simple tabular form as shown below:

	l	x'	lx'	y'	ly'
AB	10	5	50	0	0
BC	15.12	13.11	198.2	.84	12.7
CD	15.12	20.25	306.2	5.16	78.0
DE	12	29.36	352.3	6.00	72.0
EF	15.12	38.47	581.7	5.16	78.0
FG	15.12	45.61	689.6	.84	12.7
GH	10	50.61	506.1	0	0
TOTAL	92.48		2684.1		253.4

The coordiantes of the centroid, $\bar{x}$, $\bar{y}$, are found as follows:

$$\bar{x} = \frac{\Sigma l x'}{\Sigma l} = \frac{2684.1}{92.5} = 29.02 \text{ inches} \quad (7)$$

$$\bar{y} = \frac{\Sigma l y'}{\Sigma l} \quad \frac{253}{92.5} = 2.73 \text{ inches} \quad (8)$$

From this, a new coordinate system X, Y will be placed with its origin at the loop centroid.

The next step, much like the first, involves calculating moments of inertia, I_x and I_y, and product of inertia, I_{xy}, for each segment of the expansion loop and summing them in a tabular form to find the moments and product of inertia of the expansion loop.

The designation I_x indicates the moment of inertia about the X-axis, and I_y about the Y-axis. For a straight cable segment perpendicular to the axis,

$$I_{(x,y)} = \frac{l^3}{12} + la^2 \quad (9)$$

where "a" is the distance from the cable segment midpoint to the axis. If the cable segment is parallel to the axis,

$$I_{(x,y)} = la^2 \quad (10)$$

The product of inertia, I_{xy}, if found by the equation

$$I_{xy} = lxy \quad (11)$$

where x is the cable segment center to Y-axis distance and y is the cable segment center to X-axis distance.

In order to calculate the moments and product of inertia for the curved section other general equations must be followed again, for cable arc CD,

$$I_x = \frac{RK}{2}\left[\theta - 1/2 \sin 2\theta\right]_{\theta_1}^{\theta_2} - l\,(y^2 - m^2) \quad (12)$$

where all symbols are as before and y is the perpendicular distance from the X-axis to the arc centroid. These dimensions are shown in Figure 7.

$$I_x = \frac{9x2}{2}\left\{[2.412 - 1/2\,(\sin 276.4)] - [1.572 - 0]\right\} \\ +15.12\,(2.43^2 - 8.16^2)$$

$$I_x = +57.04$$

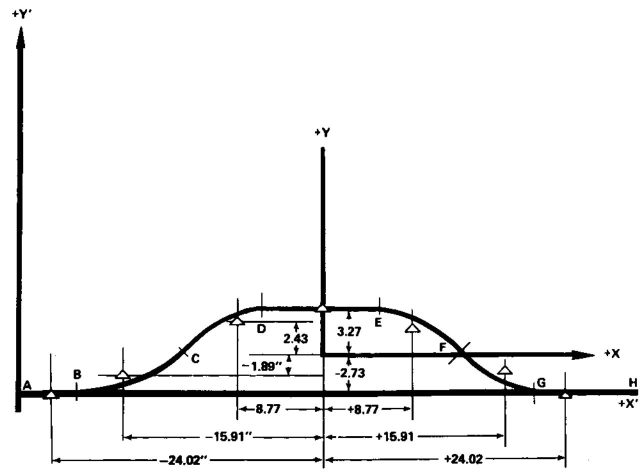

Figure 7

To calculate I_y,

$$I_y = \frac{RK}{2}\left[\theta + 1/2\ 6 \text{ in } 2\theta\right]_{\theta_1}^{\theta_2} - l\,(x^2 - n^2) \\ = \frac{9x2}{2}\left\{[2.412 + (-.497)] - [1.572]\right\} \\ +15.12\,(9.11^2 - 3.11^2)$$

$$I_y = +1358.6 \quad (13)$$

The product of inertia I_{xy} is given by,

$$I_{xy} = \left[hkR\theta - hR^2\cos\theta = kR^2\sin\theta = .5R^3\sin^2\theta\right]_{\theta_1}^{\theta_2} K \quad (14)$$

where h and k are the coordinates (h,k) of the center of the arc from coordinate system X,Y.

The products and moments of inertia of all the segments can now be tabulated and added to find the product and moment of the expansion loop.

	l	x	y	lx^2+I_o	ly^2+I_o	lxy
AB	10.0	-24.02	-2.73	+5853	+74.5	+655.7
BC	15.12	-15.91	-1.89	+4103	+12.33	+436
CD	15.12	-8.77	+2.43	+1358	+57.04	-299.4
DE	12.0	0	+3.27	+144	+272.3	0
EF	15.12	+8.77	+2.43	+1360.1	+53.36	+299.4
FG	15.12	+15.91	-1.89	+4104	+13.1	-435
GH	10.0	+24.02	-2.73	+5853	+74.5	-655.7

$I_y=22.78\times10^3$

$I_x=+557.1$

$I_{xy}=0$

From this, the forces X and Y required to compress the expansion loop .50 inches are computed.

$$X=\frac{I_y\,(\Delta xEI) + I_{xy}\,(\Delta yEI)}{I_x\,I_y - I^2_{xy}} \qquad (15)$$

and

$$Y=\frac{I_x\,(\Delta yEI) + I_{xy}\,(\Delta xEI)}{I_x\,I_y - I_{xy}^2} \qquad (16)$$

The expansion of the loop in the Y-direction from thermal expansion is assumed negligible compared to Δx and is assumed equal to zero. Therefore from equation 15,

$$X=\frac{\Delta xEI}{I_x} = 42.73 \text{ lbs.} \qquad (17)$$

$$Y=0$$

Once this is completed, the bending moments at all points in the loop can be calculated by summing moments from X and Y about the loop centroid. This was done at points A thru H and is illustrated on Figure 8. The technique now provides a full-field moment, or stress, distribution of the expansion loop under evaluation, locates the points of maximum stress, and indicates the location of nodal points. A comparison with the stress distribution found by brittle coatings in Figure 4 shows nearly identical results.

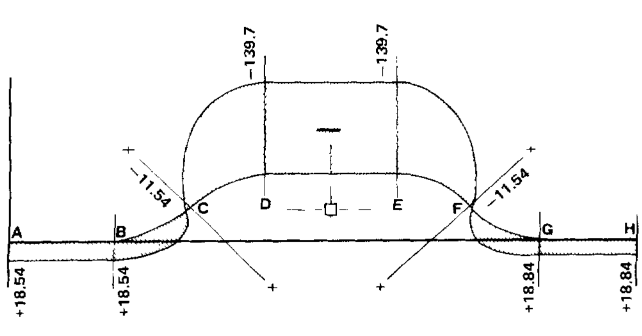

Figure 8

If desired, the strain or stress at any point may also be calculated. For an examination of the point of maximum strain, the bending moment at point $\overline{DE}/2$ is found by summing the moments at that point.

$$M_{DE}= -42.73 \text{ lb.} \times 3.27 \text{ inches} = 139.7 \text{ in-lb}$$

The strain, ε, can be readily found from equation 18,

$$\varepsilon=\frac{Mc}{IE} = \frac{139.7 \times .75}{2 \times 4.78 \times 10^{-3} \times 10 \times 10^6} \qquad (18)$$

$$\varepsilon=1096\mu\varepsilon$$

where c now is the distance from the neutral axis to the outer fiber of the tubing or assumed to be $d_o/2$.

Performing the same calculations for a drip loop type expansion loop under the same conditions and dimensions A=10, D=6, R=9, and L=0 yielded a maximum strain of $2777\mu\varepsilon$.

As mentioned earlier, the accuracy of these calculations varies with increasing displacement but it is still most helpful as a comparative technique. Theoretically, the accuracy of calculation will approach 100% as the displacement approaches zero. To place this in perspective, the measured strains at the point of maximum strain were 33% higher than those calculated at a .50 inch compression for the extended expansion loop. Reducing the displacement to .125 inches reduced the deviation to 19%, and the error continues to decrease with decreasing displacement.

The estimation of the fatigue life of an expansion loop will be the most difficult task in the design of an expansion loop; difficult in the sense of finding a meaningful technique that will be as accurate as possible with the limited amount of information that is available to the system engineer. Historically, one of the major problems of design engineering has been the necessity to work with limited and sometimes meager information because of the unavailable time to perform extended testing, or that the complexity of the variables required to obtain each individual solution makes a particular technique impractical. For these reasons, the engineer must use approximate relations. However, approximate relations can serve a very useful purpose provided the limitations of these approximations are fully recognized.

LIFE ESTIMATION

As mentioned earlier, the number of variables involved in the calculation of fatigue strengths, the stress environment, and the fatigue life of a particular structure are numerous, to say the least, and only a few of which have been incorporated into this study. Even under highly regulated and known conductions of fatigue testing of materials of a rotating beam test device, a certain amount of scatter accumulated data is anticipated.

Probably the one largest unknown variable that will affect the accuracy of the life calculations performed in this application is that of stress corrosion. The discussion of techniques of applied mechanics to compensate for stress corrosion or any involved discussion of its mechanisms is beyond the scope of this paper. It should be said though, that stress corrosion is the reduction in fatigue strength of a material which is subjected to the simultaneous environment of stress and a corrosive atmosphere. The resultant reduction of the fatigue strength is much greater than the sum of the two factors acting one at a time. Basically, the reduction in fatique evolves from a change in the surface condition of the aluminum resulting from corrosion. The presence of the roughness and minute pits generates stress risers at which very large stresses causing the material to yield. But, the true problem is not quite so simple so it is not even practical to calculate fatigue strengths for corroded samples. The only compensation that will be made at this time can be in the factor of safety applied to the overall system.

In transferring the data from the laboratory test to the field environment, the most significant contribution will be in the adjustment for stress biasing. Looking at Figure 9[2], the three general types of fatigue stress are illustrated. Where

σ_{min} = minimum stress	σ_m = mean stress
σ_{max} = maximum stress	σ_r = stress range
σ_a = stress amplitude	σ_s = static stress

FATIGUE STRESS TYPES

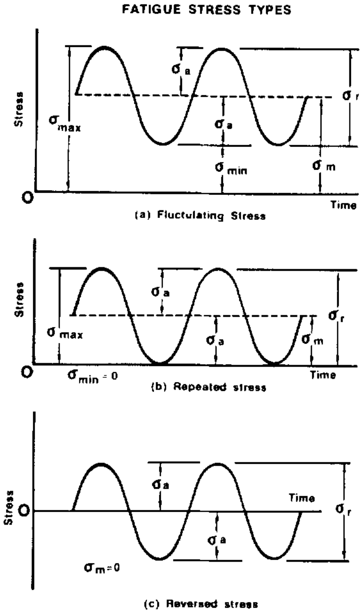

Figure 9

Those tests conducted to determine the number of cycles to failure versus excursion were conducted in reversed stress, σ_m=0, representing the conditions of repeated and fluctuating stress is done by adjusting for an increasing mean stress, σ_m.

In Figure 10, a representation of the environmentally induced strains is shown in simplified. The seasonal temperatures have been simplified to two temperatures with a transitional season between them. Superimposed on the seasonal temperature are shown the daily temperature changes. The sum of the seasonal and daily temperature change is identified as the theoretical strain curve. The strains within the cable and expansion loop will follow this curve and their magnitude will vary with that curve and the difference between the mean annual temperature and installation temperature. It can be seen that if the mean annual temperature and installation temperature were identical, the mean strain would be zero and the strain cycle would closely approximate the laboratory testing. The importance of installation temperature now becomes apparent. If, for example, the cable was installed the coldest day of the year, the mean stress on that day is zero, but for every remaining day the mean strain would increase as would strain amplitude. During the warmest day of the year the maximum stress will be reached. The same is true in reversing the situation.

ENVIRONMENTAL STRAIN PROFILE

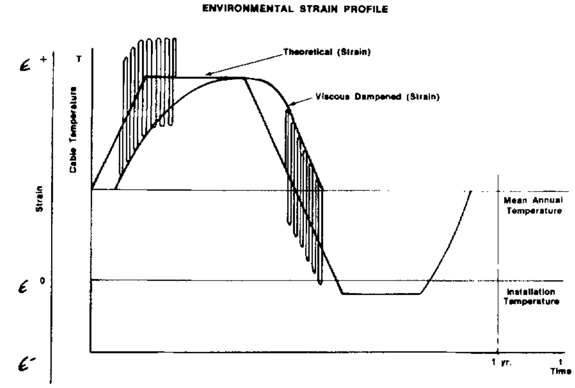

Figure 10

The viscous dampened curve in Figure 10 represents the strain in an expansion loop which is installed in a system where the cable is tightly lashed. Because of lack of freedom of movement, the daily temperature excursions are dampened out, but the seasonal excursions still exist with a given amount of lag. What actually exists in the field at present, would range over the entire spectrum between the theoretical and dampened curves. For design consideration, the theoretical curve should be used since its higher number of cycles represents the worse condition.

The reduction in life, resulting from stress biasing, of course varies with the magnitude of bias, and it is understood that not all conditions are as severe as previously illustrated. To evaluate the reduction in life at any level of

[2]Joseph Edward Shigley, Mechanical Engineering Design. New York: McGraw-Hill, 1972, second edition, pg. 269.

bias, the Langer modified Goodman Diagram[3] is useful. This is illustrated in Figure 11. In this diagram, the curve to the left of the vertical axis is the S-N curve constructed from the composite of strain measurements made and the fatigue life testing. It is plotted stress versus number of cycles to failure (non-logarithmically). The right side is constructed by drawing a line from the yield stress, σ_y, plotted on both the ordinate and abscissa. To find the modified life for a particular alternating stress at a level of bias, A, calculate or measure the strain amplitude associated with the alternating strain and compute the corresponding stress. This must be done from the stress-strain curve of the aluminum outer conductor and not taken from conventional wrought aluminum data. Corresponding to this stress, will be the life at that stress, N, for a mean stress of zero. Next, draw a line from the alternating stress valve, σ, on the ordinate to the ultimate tensile strength of aluminum, σ_u, on the abscissa. For the particular mean stress, σ_{mA}, associated with the difference between the installation temperature, and the mean annual temperature and seasonal temperature, point A can be located by the intersection of σ_{mA} and line $\overline{\sigma\sigma_u}$ or $\overline{\sigma_y\sigma_y}$ whichever is less. The modified alternating stress, σ_{aA}, for point A, then corresponds to the modified life N_2 on the S-N curve. The reduction in life is the ratio of N_1 and N_2.

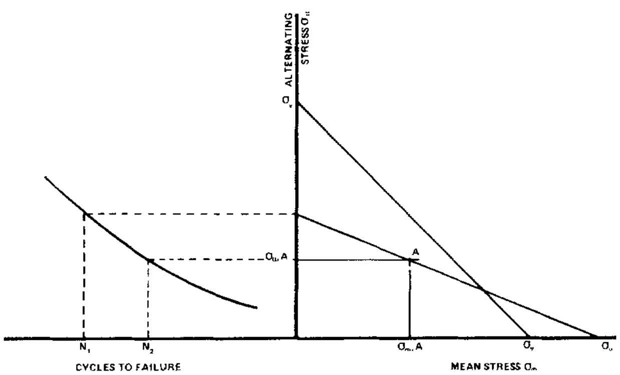

Figure 11

The final step in determining the life of the expansion loop is the combining of all the various different lines corresponding the number of various bias level calculations which have been made, and the number of cycles occuring at each level. One of the most popular theories for accomplishing this is the Palmgren-Miner Theory.[4] This theory states that the portion of the total cycles to failure applied at one stress level plus that portion of the total number of cycles to failure applied at any other stress equals a constant. This constant, K, is found experimentally and normally ranges from 0.7 to 2.2. For simplicity K=1 is most often used. Stating this theory mathematically,

$$\sum \frac{n}{N} = 1$$

where n is the number of cycles applied at a single stress and N is the number of cycles to failure at that stress.

The results from this technique, as with the others, must be qualified. In evaluation with experiment, the theory often fails to be in agreement; however, other more accurate and complex theories are available with improved correlation. The use of these improved theories is not required at this time in light of what is to be gained and the number assumption already operating. Again the use of this theory is intended as an improved guideline and not for use in obtaining absolute valves.

CONCLUSION

The problems associated with fatigue in coaxial cable is a highly complex problem which will require much more testing before the problem can be comfortably handled, if ever. Until that time, the engineer must take full advantage of the limited data availability to insure the maximum reliability of their system. By applying many of the experimental and analytical techniques already available, and large factors of safety, the life of present cable systems can be vastly improved.

ACKNOWLEDGEMENT

The author wishes to acknowledge the contributions of Mr. Gordon Huffman, Environmental Engineer, Superior Continental Corporation, R & D, Hickory, North Carolina, for his assistance and development in much of the life testing presented.

[3]S.S. Manson, Thermal Stress and Low-Cycle Fatigue. New York: McGraw-Hill, 1966, pp.177-180.

[4]A. Palmgren, DieLebensdauer von Kugellagern, ZVDI, pp. 339-341, 1924; M.A. Miner, "Cumulative damage in fatigue," J.Appl. Mech., vol. 12, Trans ASME, pp. A159-A164, 1945.