ON THE REALITIES OF NON-LINEAR DISTORTION IN CATV AMPLIFIERS

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INTRODUCTION

Look at the following statements, and try to mark each one as true or false:

- Triple beats and third-order cross modulation increase by 2 dB for each dB increase in carrier level.
- When the input levels are the same on all channels, the third harmonic component is 15.5 dB weaker than the triple-beat component.
- The cross modulation in an N-channel system is 20log(N-1) dB higher than that measured in a 2-channel test.
- In a wide-band multi-channel system, where the spacing between any two carriers is an integral multiple of a basic frequency, the worst triple-beat interference is in the channel at the center of the band.

We have all been using these and similar rules as a basis for system design and for evaluating equipment, and the fact that we are still in the business shows that there must be at least some truth in each of the above statements. But we have to recognize that, although the statements are correct deductions from certain assumptions, their truth depends on the validity of these assumptions:

1. The output y from a single amplifier, or from the cascade that forms any point-to-point portion of a transmission system, can be represented by a power series of the input x,

$$y = k_1x + k_2x^2 + k_3x^3 + k_4x^4 + k_5x^5 + \dots$$

   where the k's are defined as independent on the input x.

2. The distortion that limits the operation of an amplifier, and the length of a cascade, is due to the third term in the series. (The effect of the 2nd and higher even order terms can be reduced by push-pull circuitry. The 5th and higher order odd terms are assumed small enough to be disregarded at the levels at which the amplifiers operate).

3. The coefficients k are real numbers and do not depend on the frequencies of the signals.

The only legitimate way to decide the truth of any statement is to subject it to a direct test. If the results are not in complete accord with the statement, we have no choice but to reevaluate the basic assumptions and modify them to agree with reality.

This paper is a report on two experiements that I recently conducted while studying the non-linear distortion in wide-band transistor CATV amplifiers, and my interpretation of the results.

I. THE CASE OF THE ILL-BEHAVED AMPLIFIER

The Motive

I think that we will all agree that the first statement, about distortion vs. level, is not universally true. We have had experience with amplifiers that do deviate from the 2:1 law, and we have been rewarding those that follow this law by the mark of "well-behavior". The "ill-behavior" of amplifiers really poses an intriguing problem, because a 3rd order effect must by definition follow a strict 2:1 law. For example, let a be the amplitude of each of 3 carriers at frequencies f_1, f_2 and f_3, producing a triple-beat at the frequency $f=f_1+f_2-f_3$; the amplitude of the beat would be k_3a^3, and the carrier-to-intermodulation ratio is $1/k_3a^2$. In a dB/dB plot, this is a straight line with a 2:1 slope. How then can the amplitude of a triple-beat deviate from this rule?

Let us look more closely at the problem. We compute the frequency f of the beat, and measure the distortion product which is generated at that frequency. Now it is true that the particular triple beat of the three given carriers can occur only at that frequency and nowhere else, but the converse is not true at all: it does not follow that what we observe and measure at that frequency must be that triple-beat and nothing else. The mere fact that the measurement deviates from the 2:1 law indicates that we must be measuring something else.

I suggest that the deviation from a 2:1 law is due to the effects of higher order intermodulation products, whose magnitudes (at the levels at which the amplifier is tested) are close to those of the 3rd order effects. To justify the assumption, I shall proceed in 3 steps:

1. Prove that higher order terms can contribute distortion products at <u>precisely</u> the same frequency <u>f</u> of the triple-beat.

2. Compute the interaction between third-order and higher-order distortion products occurring at the same frequency.

3. Design an experiment to produce a single isolated triple-beat interacting with different amounts of higher order distortion, and compare the measured results with the theoretical computations.

<u>The Method</u>

A 5th order intermodulation product is generated by the interaction of 5 "parent" carriers; its frequency is a sum and/or difference combination of the frequencies of the carriers, and its amplitude is proportional to the product of their amplitudes. Any given carrier can play the role of one or more of the parents, so that 5th order effects can be generated even if there are less than 5 carriers. In particular, an amplifier processing 3 carriers can produce 5th order products whose frequencies can be computed by A+A-A+B-C, A+B+B-B-C and A+B+C-C-C; this is precisely the same frequency as that of the 3rd order A+B-C combination, no matter what A, B and C are.

If all carriers have the same amplitude <u>a</u>, the amplitude of a 5th order product is proportional to a^5, and the carrier-to-intermodulation ratio would be proportional to $1/a^4$. Plotted on a dB/dB scale vs. carrier amplitude, this would be a straight line with a 4:1 slope. When a 3rd order and 5th order effect produce a product at the same frequency, the resultant beat would be the sum of the two distortion products. At any one given signal level, one or the other component may predominate; but, no matter how small the 5th order coefficient k_5 may be compared to k_3, the 4:1 line <u>must</u> intersect the 2:1 line and overtake it at some carrier level. Fig. 1 shows some of the possible combined characteristics. All the curves start as a 2:1 line on the left, and end up getting close to the 4:1 line on the right; the shape of the curve in the transition region will depend on the <u>phase</u> between the 3rd

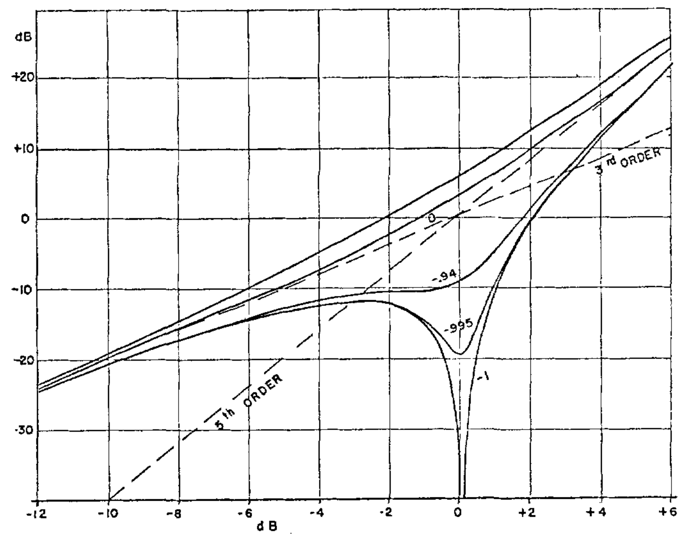

Fig. 1

order and 5th order products. The parameters marked on the curves are the cosine of the phase angle, and a positive value will result in a montonically increasing curve. A cosine of -1 produces a sharp null at the crossover , and any value between -.95 and -1 will produce a curve with a minimum point. The curve resulting from a cosine of -.94 has a flat portion, where the intermodulation is independent of level.

Any curve of this family can be completely characterized by the position of the 2:1 and 4:1 lines, and the phase angle between the two components. Fig. 2 shows a convenient set of such parameters:

X dB - the value of the 3rd order component at an arbitrarily selected level (in this case, 50 dBmV),

Y dBmV - the level at which the two lines intersect, and

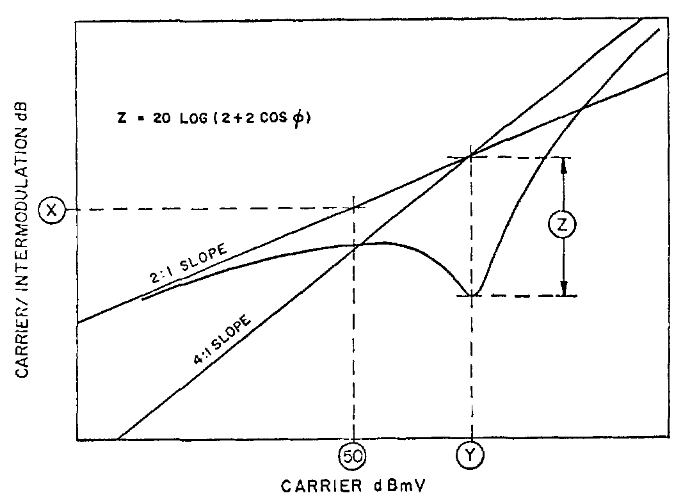

Fig. 2

Z dB - the vertical displacement of the curve from the point of intersection. (This parameter, whose value may vary between + 6 dB and $-\infty$, indicates the phase between the two components.

The curves of Fig. 1 should look very familiar to anyone who has done any measurements of cross modulation or triple-beat. I have successfully matched the curves to the records of numerous measurements made over the years in single transistors and complete amplifiers. However, some interesting features of the interaction can best be studied in a specially designed experiment, which I shall describe here.

In a 12-channel system, with the standard VHF assignments, there is a single "isolated" triple-beat at 119.25 MHz, produced by the intermodulation of the picture carriers of channels 6, 7 and 13. Because channels 5 and 6 are off the uniform scheme of 6 MHz steps, no other triple-beat falls at that frequency, (the closest ones are at 113.25 and 139.25 MHz). It is therefore possible to study the behaviour of a single triple-beat in a multi-channel system. The existence of 5th order components that may contaminate it does not depend on the regular channel spacing, and as the band is gradually filled in (from 3 to the full 12 channels), the amount of 5th order interference, and its effect on the measurements, will keep increasing.

The Clues

The ratio of the measured intermodulation at 119.25 MHz to the carrier level, for various carrier levels and carrier combinations, is shown in the table of Fig. 3. The carrier combinations used were:

MEASURED INTERMODULATION AT 119.25 MHz

IN dB BELOW CARRIER LEVEL

dBmV EACH CARRIER	6, 7, 13 ONLY	12 ADDED	11 ADDED	10 ADDED	9, 8 ADDED	2, 3, 4, 5 ADDED
48	68.5	68.5	68.5	69.5	68.5	68
50	64.5	65	64.5	65.5	65.5	64
52	60.5	61	60.5	62	61.5	59
54	56.5	57.5	56.5	57.5	58	54
56	52	53.5	51.5	51.5	52	47
58	48	49	46	43.5	46	40
60	43.5	42.5	39	35.5	38.5	33

NOTE: THERE IS ONLY ONE COMBINATION OF STANDARD VHF CARRIERS THAT PRODUCES A TRIPLE BEAT AT THIS FREQUENCY

211.25 − 175.25 + 83.25 = 119.25

(13) (7) (6)

Fig. 3

(a) Channels 6-7-13.
(b) Channels 6-7-12-13.
(c) Channels 6-7-11-12-13.
(d) Channels 6-7-10-11-12-13.
(e) Channels 6-7-8-9-10-11-12-13.
(f) All 12 VHF channels.

A cursory examination of the table, by column, will show that in all cases the intermodulation increases with level, although not on a uniform 2:1 rate. Scanning the table horizontally we are due for a surprize: adding channels does not necessarily result in increased intermodulation at the same carrier level. Before you try to dismiss this as a possible error of measurement, let me add that this fact was demonstrated and checked out very carefully. For example, the beat was observed on a spectrum analyzer with all carriers at 52 dBmV, and when the carrier of channel 10 was switched on, going from condition (c) to (d), the level of the beat dropped a very visible 1.5 dB.

When the measurements are plotted in the conventional manner (Fig. 4), they are seen to curve away up from the straight 2:1 line, but the curves are too clustered to show the effect in detail. The same curves are redrawn in Fig. 5 on a distorted grid, in which a pure 2:1 3rd order characteristic would appear as a horizontal straight line, and any deviation from that line is accentuated.

The measurements were analyzed by computer to find how well they could fit a curve of the family in Fig. 1, described by the paramenters of Fig. 2, with these results:

Carrier Combination	(a)	(b)	(c)	(d)	(e)	(f)
Parameters of matching curve						
X dB	64.6	64.2	64.3	64.5	64.5	64.5
Y dBmV	65.9	58.6	57.4	54.9	56.4	54.3
Z dB	4.5	-0.1	2.1	1.2	1.0	3.1
cos Ø	.41	-.51	-.19	-.35	-.37	.01
Match between measurement and curve						
Peak deviation dB	.20	.18	.15	.21	.59	.36
Rms deviation dB	.10	.11	.09	.14	.37	.26

Observe the last two linesin the table. Taking into account that the measurements were made to the closest 0.5 dB (see Fig. 3), the fit must be considered excellent.

The most interesting result of this experiment is in the "X" values, which are practically identical for all 6 measurements. This means that all the different curves of Fig. 4 or 5 have the same 3rd order component, the difference between the curves being due only to the amounts of 5th order contribution to the measured values. The 119.25 MHz beat was chosen to check this point, because no other combination of any 3 of the 12 channels could contribute a 3rd order intermodulation at this frequency.

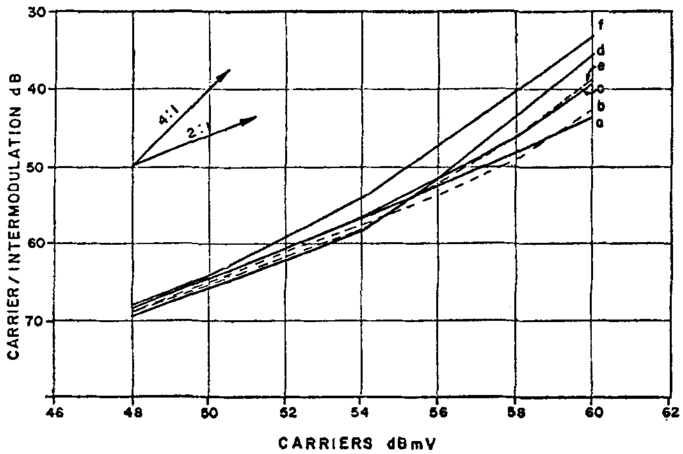

Fig. 4

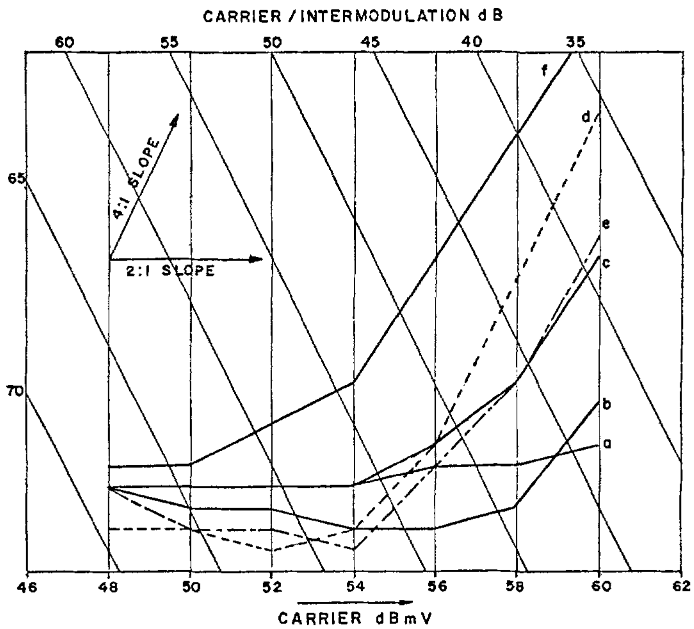

Fig. 5

The "Y" value of the table indicates the magnitude of the 5th order effect, and Fig. 6 shows the 5th order contribution of the various carrier combinations. We see that even in case (a), with only three carriers present, the 5th order effect at 60 dBmV is enough to cause a deviation of 1 dB from the 2:1 line. Adding channel 12 increases the 5th order contribution by 14.5 dB, moving the intercept 7.3 dB to the left; however, at the levels measured, curve (b) is almost everywhere below curve (a)! This is explained by the cosine factor, which is positive in (a) but negative in (b). Curve (a) results from adding a small amount of 5th order to the basic triple beat; curve (b) results from a much higher contribution of 5th order being subtracted.

When channels 11 and 10 are added, the 5th order component keeps increasing, with the intercept moving to the left. But when channels 8 and 9 are added, we suddenly observe the 5th order line receding. Although 8 channels can produce many more 5th order combinations than 6 channels, the relative phases of the contributions must be such that the resultant has a lower magnitude.

The Verdict

The experiment shows that it is impossible to measure a pure triple-beat. The measurement will always be contaminated by higher order distortion products that fall exactly at the same frequency, and this is the reason for the deviation of the measured intermodulation from the simple 2:1 law. (A similar argument can be made about the measurement of a distortion of any order).

The accumulation of beats of the same order, and the interaction of beats of different orders, have to be computed by complex number arithmetic (amplitude and phase). This means that the coefficients in the power series are frequency dependent complex numbers.

No amplifier deserves to be described as "well-behaved" or "ill-behaved". Unless the amplifier characteristic is a pure 3rd order function, it must deviate from a 2:1 law at high enough levels, because a 2:1 line and 4:1 line must intersect somewhere. Unless the amplifier characteristic totally lacks the 3rd order term, any amplifier will behave "well" if the levels are low enough.

Is anything left from the notions we held until now? Fortunately, it seems that we can still extrapolate the measured intermodulation and cross modulation to lower levels, if we are sure that we have measured it at levels below the crossover with the 5th order effects. If

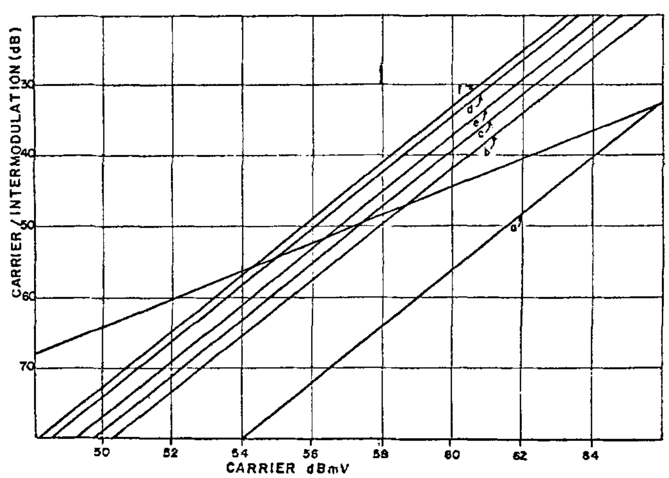

Fig. 6

the measured characteristic follows a strict 2:1 line over a range of, say, 20 dB, we can safely extrapolate to any lower level. Even if the measurement is made where there is 5th order interaction, we can be sure that there will be no anomalies at lower levels.

II. THE CASE OF THE PILED-UP BEATS

The Motive

In a transmission system with regularly spaced carriers, the triple-beats (of type A+B-C) tend to pile up close to the carriers, and the number of beats in any channel increases much faster than the number of channels. If we assume that the number of separate beats that occur at a given frequency also represents the amplitude of the resultant beat, we may very easily be discouraged from increasing the number of channels in the system. While in a standard 12 channel system the largest pile-up is 19 beats (in channel 10), it increases to 107 (Channel E) in a 21 channel system, and 202 (in channels F, G and H) when the system carries 27 channels. The pile-up is worst in the channels at the center of the band, as can be seen in column (a) of the table below, computed for 21 channels (channels 5 and 6 at 2 MHz higher than the standard assignment):

Channel	(a)	(b)
2	59	59
3	67	66
4	70	69
5	76	76
6	78	78
A	96	91
B	101	95
C	104	97
D	106	100
E	107	101
F	106	101
G	104	98
H	103	96
I	102	95
7	100	94
8	99	94
9	97	93
10	98	91
11	89	87
12	84	83
13	76	76

The purpose of my second experiment was to verify this phenomenon.

The Method

The combination of over 100 beats that occur at approximately the same frequency is very difficult to measure. If the carriers are spaced precisely 6 MHz apart (by phase-lock techniques), all the beats occur at the same frequency, and their resultant is a stable, measurable signal; however, its frequency is exactly that of the carrier of the channel where they occur. I therefore modified the experiment by removing one carrier at a time, and measuring the resultant beat which was generated by the remaining 20 carriers at that frequency. Column (b) of the table shows the number of triple-beats that pile up at each carrier frequency under these conditions. The measured beats are plotted in Fig. 7 for two different values of the carriers. (Only the discrete points, shown as circles and crosses, are significant. The lines connecting the points are included only to bring out the pattern).

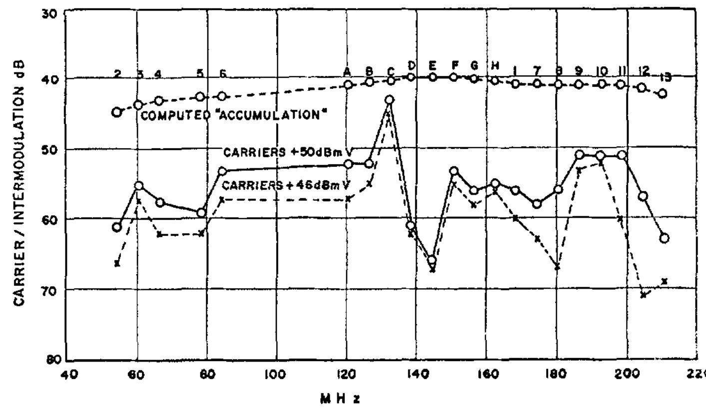

Fig. 7

The Clues

The points on the upper curve, marked "accumulation", represent values proportional to the numbers in column (b) of the table. The curve is intended to show the variation that would be expected if the amplitude of the resultant beat were proportional to the number of triple-beats that combine to produce it. (For the purpose of the discussion, only the shape of the curve is significant, but not its vertical placement in relation to the other curves).

It is clearly evident that there is no simple correlation between this gradually changes curve, and the results of the measurements which show significant up-and-down variations between adjacent channels. The combined beat in channel E, which is supposed to be at the peak of the curve, is almost at the lowest measured point. The interference in channel E, which is a combination of 101 different triple-beats, is 23 dB lower than the combined 97 beats in channel C. The measurements also show that a 4 dB change in the level of the carriers caused the accumulated beat in some channels to change by 1 dB only, and by 14 dB in other channels.

The Verdict

Given any combination of carriers, the number of beats that fall into any channel can easily be computed. We may use exact formulas of combinatorial theory, or try for approximations such as $0.162N^{2.21}$, and express the result in logarithmic form (as "voltage dB" or "power dB").

Unfortunately, these numbers have no correlation to the amplitude of the resultant beat.

It can easily be verified that the magnitude of any particular triple-beat depends on the frequencies of the 3 carriers that generate it, so that the 100 or more beats that accumulate at one frequency are not of the same amplitude. This experiment forces us to conclude that the different triple-beats must differ in phase as well as in amplitude. Consequently, there is no way to predict the measured results plotted in Fig. 7 short of measuring (or computing) the amplitudes and phases of each one of the hundreds of possible triple-beats that combine to produce the total effect.

CONCLUSION

The simple series representation for amplifier distortion, assuming real, frequency-independent coefficients, seems to be invalid for transistor amplifiers as used in CATV. Direct measurements force us to the conclusion that the coefficients are functions of frequency, and have to be complex numbers. (Mathematically it means that the Volterra series, rather than the Taylor series, is to be used in the representation).

In view of this, some of the statements at the start of this paper are neither true nor false, but meaningless. For example, how can you compare "third harmonic" to "triple beat"? The third harmonic of which carrier is supposed to be 15.5 dB below the triple beat of which 3 carriers? The cross modulation in a 12 channel system is supposed to be 20log(11) dB above that measured with which 2 channels?

If we do not know the exact frequency dependence of the coefficients, we cannot predict the distortion of one combination of carriers from measurements with another combination. There is no way to predict a combined effect unless we know the amplitudes and phases of all the components in the combination. If we want to approve an amplifier for the trunk of a 27 channel system, we have to measure the distortion produced by the same 27 channels in the amplifiers, and not extrapolate from 12 channel behaviour (unless we want to accept a "worst-case" estimate which will be completely unrealistic). I would not be surprised to find that an amplifier which was operating well in a 27 channel system, is found to have too much distortion in one or more channels when it is tested for use in a 20-channel system.

The effects of 5th and higher orders are far from being negligible at the levels at which CATV amplifiers are operated. The higher-order distortion products are the cause of the deviation from "well behavior". Depending on the phase between the 3rd order and the higher-order products, the effect may be a slight variation from a strict 2:1 law, or result in deep minima and sharp nulls. However, it is possible to extrapolate the measured distortion to lower levels, provided we have properly extracted the 3rd order component from the higher order effects.

The complex-number, frequency dependent mathematical model is more complicated than the simplified assumptions that we have been used to. But when our computed predictions, based on any one model, seem to be at variance with the reality brought out by measurements, there is no question as to which of the two should be modified to conform to the other.