AN ANALYSIS OF SYSTEM PLANNING

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The traditional approach to the building of a CATV system is to erect an antenna at a convenient location and distribute from the antenna location to all of the subscribers the received channels by means of an all-band cable system. As CATV penetrates the larger cities and as bi-directional transmission of signals becomes a reality, the trend is toward the segmentation of systems into areas served from separate hubs connected by various types of cable or microwave links. Within this new approach are contained so many alternatives for types of hubs, connecting links and combinations of forward and return systems that system planning has become a task of bewildering complexity involving the owner's objectives and the relative costs of various methods of implementation. This paper analyzes the general problem and describes a computer program developed as a tool to permit analysis of the system planning problem. The analysis permits a recommended solution to be found by the computer given the objectives, costs and general physical data of the geographical area or permits an analysis of preferred methods of system configuration. The analysis involves the use of a new family of mathematical operators developed by the author, called "COM" operators, which simplifies some of the mathematical problems involved.

1.0 Introduction

Today most prospective builders of new CATV systems have a fairly detailed concept of what they want their system to do. Due to FCC requirements, franchise requirements and market requirements they know what the channel capacity of the system must be for present and future needs and what system performance is required. Additionally, they are planning some special services that will use the bi-directional capability of their new system. However, the area of knowledge which has not yet been extended to a major portion of the industry is the details of assembling a complex system in a relatively large metropolitan area to meet the desired system operating and performance objectives.

2.0 Need for Analysis of System Planning

It is not uncommon for an owner to express surprise and dismay when informed it is not possible to build an all-band system extending from his headend location that will serve all of the subscribers with satisfactory signals. It is readily apparent that some kind of hub concept is necessary, but a more complex system rapidly produces more complex problems and rapidly increasing costs. To illustrate, a typical system configuration is shown in figure 1.

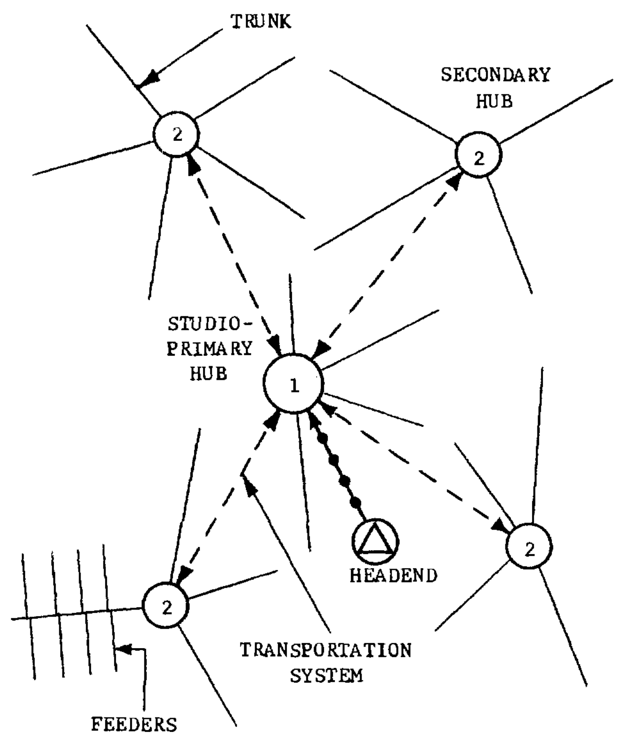

FIGURE 1
TYPICAL SYSTEM CONFIGURATION

2.1 Elements of a System

The center of interest of the system shown in the figure is the primary hub, centrally located in the area. The owner's studio is located in the primary hub along with the switching equipment to control the distribution of signals to the system. A transportation cable system carries signals into the primary hub from the headend, located remotely for convenience of off-air signal reception, and other transportation cable systems carry signals to and from the secondary hub sites. Emanating from the secondary hub sites are bi-directional trunk and feeder systems for distributing signals to and collecting signals from the subscribers. As may be practical microwave links can replace some cable transportation system links.

Of particular interest to us are those specific elements of the system that contribute to signal degradation. These elements are the processing equipment in the headend and hub locations, any conversion equipment used in connection with the transportation systems, the amplifiers in the cable systems and any microwave equipment used.

2.3 Conventional System Operation

The most conventional method of operating the system would be to receive all off-air channels at the headend and transmit them to the subscribers via the primary and secondary hubs. Any signals originated at subscriber locations would be carried to the primary hub and re-distributed to the subscribers along with the off-air channels. The problem with this arrangement is that all of the channels are carried from the primary hub on the hub-to-hub cable transportation links. Since the degradation in a cable system is proportional to the number of channels carried, the degradation in the transportation systems may make it costly or even impossible to design the trunk and feeder system to meet the overall system performance objectives.

2.4 Alternative System Operation

As an alternative only the distant channels may be received at the headend location with strong local signals received with less expensive antennas located at the secondary hubs. Additionally, not all signals originated at subscribers may be of interest to the entire area, and the signals of purely local interest could be intercepted and redistributed from the secondary hubs. The resultant reduction in the number of channels carried on the transportation systems and the corresponding improvement in performance permits a less costly basic design. However, the above alternatives introduce a new cost factor; i.e., a requirement for channel processing equipment at the secondary hubs.

Additionally, by dividing the system into a larger number of areas; i.e., more hubs, the amount of noise degradation collected at each hub in the return system is reduced, but more transportation systems are required to carry the signals from the collection points to the primary hub.

Another simple way of improving transportation system performance is to use a dual cable system. But how does the owner know which approach is the least costly? The amount of trial and error required to optimize the performance of any one approach is so formidable that it presents obstacles to producing a proposal within the time frame required by a customer. But when a change in system performance of a few dB may mean a change of several hundred dollars per mile in overall system cost, it is mandatory that the system builder carry out a system planning program that involves analyzing many different approaches to the implementation of his system objectives.

3.0 Solution to System Planning Problem

The purpose of this paper is to provide the mathematical analysis which makes possible the rapid evaluation of alternate types of systems. In particular, this analysis provides the basis for a tool for the system planner. Based upon this analysis a computer program has been written by the author which permits the system planner, sitting at a terminal of an ordinary commercial computer time sharing system,to selectively specify alternate system configurations to the computer, which determines optimum performance and automatically investigates some of the alternatives.

The analysis contained in this paper was specifically designed to provide the basis of a computer program concept that entailed the following steps:

(1) Apportion the strand mileage between the specified return hubs such that for any hub the performance of the combination of the return line extender, trunk and transportation amplifiers is maximized and is the same for all hubs. This process requires the finding of the best allocation of performance degradation between the several elements.

(2) Find the combination of forward transportation and trunk systems which has the worst performance. This process also requires the finding of the best allocation of performance degradation as above.

(3) Allocate performance degradation between the sub-systems of (1) and (2) for best performance.

(4) Find the trade-off of performance degradation between the sub-system of (3) above and the feeder system such that the system noise and cross-modulation performance specification is just met with the maximum possible levels in the feeder system.

(5) If there is no solution within the constraints imposed upon the system, the computer would increase the number of return system hubs until a solution is found. A maximum number of hubs may be imposed to avoid absurd answers. When the computer "creates" new hubs to increase the

total number, average values are used for the equipment densities and transportation system cascades in the added hubs.

4.0 Details of Analysis

4.1 Notation & Fundamentals of COM operators

To begin the analysis the reader should refer to Appendix 1, which gives an explanation of the notation used in this paper. Secondly, in Appendix 2 are shown the relationships for "COM" operators. These operators are used to express in simple notation the combining of noise or cross-modulation. For example, if amplifier A has a carrier-to-noise ratio of Na and amplifier B has a carrier-to-noise ratio of Nb, the carrier-to-noise ratio of the two amplifiers operating in cascade is by definition Na #p Nb; i.e., Na combined with Nb on a power basis. If we let

Na #p Nb = Nt,

then by definition Na = Nt p\ Nb

Appendix 2 summarizes the above for cross-modulation as well as noise and lists other relationships that simplify derivations used herein. Derivation is accomplished in every case by straight forward substitution of the basic logarithmic/exponential expression into the stated relationship.

4.2 Maximum Performance

The first derivation that has a direct bearing on system performance is contained in Appendix 3. The expression obtained in this appendix calculates the difference in output level which produces the maximum performance for two units operating in cascade. Maximum performance is defined as the maximum value of X + 2N. The quantity X + 2N is independent of operating levels, assuming that relative level is constant, since (in an ideal system) an increase in all levels by 1 dB causes the carrier-to-cross modulation ratio to decrease by 2 dB and noise to improve by 1 dB. X + 2N is herein defined as the Performance Factor, PF.

4.3 Apportionment of Strand Miles Between Hub Systems.

With varying distances between the primary hub and the secondary hubs and with varying sizes of system fed from each secondary hub there can be a substantial difference in performance in the return system for each return collection and transportation sub-system. Rather than selecting the worst case and basing the calculations on this worst case the system planner can improve his overall system performance by allocating the strand mileage between the sub-systems served by each secondary hub so that each sub-system has the same performance. In other words, since the return performance of all segments is made the same (by adjusting their size) the "worst case" is improved to be the average return system performance. In Appendix 4 the performance factor

PFrt = Xrt + 2Nrt

is derived for the average return system performance as a function of the sub-system strand mileage, the equipment densities and the equipment performance parameters. With the expression obtained, a linear programming technique can be used that finds the strand mileage for each secondary hub system such that the sum of the strand mileages of the hub systems is equal to the total system strand mileage and the performance factor is the same for all hub systems.

4.4 Calculating Feeder Performance

Finally, in Appendix 5 a relationship is derived that permits the system levels to be found that just meet the system specifications. The expression involves the feeder cross-modulation, Xf, the system specifications, the performance factor of the feeder and the performance factor of the rest of the system.

Since a direct solution for Xf is not easily obtained, it was decided to use an iterative method for obtaining a solution. The method is illustrated in figure 2.

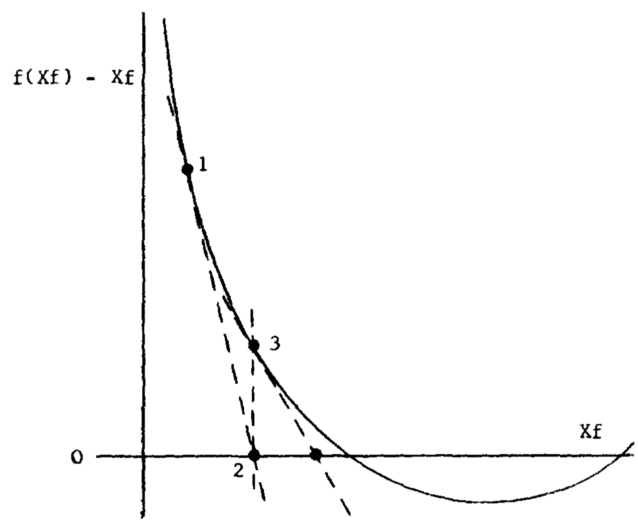

FIGURE 2
SOLVING FOR FEEDER CROSS-MODULATION, Xf

1. At point 1 the slope of the line tangent to the curve was found by incrementing Xf by a small amount.

2. The point at which the tangent line intercepted the "0" value of f(Xf)-Xf was calculated. (point 2).

3. Using the value of Xf so obtained a new point 3, was found on the curve.

4. The process was repeated until a value of f(Xf)-Xf less than an arbitrary value δ was found.

5. By always using negative increments in step 1 above, it is assured that a change in sign of the slope of the tangent line indicates that there is no solution.

After some experimentation with this method it was found that the largest number of iterations required to find a solution occur when the system specifications are near the maximum possible performance. It was found that with a value of $\delta = 0.01$ dB and the system specifications within 0.0001 dB of the maximum system performance, only 6 iterations were required to find a solution or determine that a solution was impossible. Typically a solution or non-solution was found in 2 tries with less severe constraints.

5.0 At the Computer Terminal

To use the computer program you would dial up the computer from your terminal and load the program. The computer types out on your terminal READY! Now you have a selection of things which you may type.

The program is interactive, i.e., it executes a command entered from the terminal and then prints READY! again and waits for the next command. The commands available are:

FILE — causes the computer to read a computer data file which must be constructed before loading the program. The file contains for each hub the cascades, the equipment densities, the equipment noise figures output capabilities and gains for each sub-system, the number of return system hubs with the equipment densities, trunk cascades and transportation system cascades and proposed strand mileage.

PACK — Loads a specified number into the same element in all hubs. For example, permits assigning a new value to the return transportation system amplifier output capability in all hubs.

DIF — Establishes the difference in output level between bridger amplifiers and line extender amplifiers.

FIX — Freezes the levels in the feeder at their current value.

MIN BR — Establishes a minimum bridger amplifier output level.

MAX RH — Sets the maximum number of return system hubs permitted.

NHUBS — Sets the number of return system hubs.

RHUB — Permits performance degradation for the return system hubs to be specified.

FHUB — Permits performance degradation for the forward system hubs to be specified.

H END — Permits performance degradation for the headend system to be specified.

MIC — Permits specifying a fixed performance degradation for a transportation system.

SPECS — Establishes the system noise and cross-modulation specification at desired values.

FWD HB — Displays the number of the worst case forward hub system.

MAX — Finds maximum possible performance of system as presently configured.

RUN — Apportions strand mileage between return hub systems and calculates all system levels to meet the system noise and cross-modulation specifications. If no solution is possible within the limits imposed, a MAX command is automatically executed.

The first step is to issue the FILE command to load the basic data, then the other commands are typed on the terminal keyboard to establish the system parameters. The function of most of the commands is self explanatory. In each case the system planner types the command exactly as shown. The details of the specific item, i.e., the output capability, etc., and the relevant data, are input by the system planner after the computer prints out a message specifying that they are to be typed on the keyboard. No knowledge of computer operation is required by the user.

In particular it has been found that the strand mileage calculated by the computer for a particular return hub system may not be suitable due to the physical constraints of the geographical area. If a smaller number of strand miles must be used, the performance of that particular sub-system will be better than average, but that of the rest of the system will be poorer, since the strand mileage for the rest of the system will be increased. In this instance the strand mileage of the smaller than calculated hub system is subtracted from the total mileage, and the remainder of the system is re-processed as a separate system.

If a larger than calculated strand mileage must be used, the performance of this larger sub-system will be poorer than the remainder of the system and the performance of this sub-system becomes the worst case and is then processed as an independent system.

If the computer determines that a larger number of return system hubs is required than originally desired, the system planner may change the minimum feeder level limits, change the output capabilities of the amplifiers in the transportation systems (different number of channels) or enter specific information to include the added number of hubs.

To evaluate the cost of the various configurations it is necessary to have available cost per mile information on the types of transportation systems to be used, the cost of active hub sites, including buildings and equipment and the distribution system costs. Clearly, in the planning stage of a system close communication is required with the system owner to determine what locations are available practically. The distribution system cost can be based upon a sample design keyed to a fixed set of amplifier output levels.

The above analysis and associated computer program is providing us with a much deeper insight into the general area of system configuration to the point that questions have arisen that were not previously considered. Never-the-less in order to adequately analyze the alternatives possible in a large CATV system it is absolutely essential that tools be available that permit rapidly finding the best means to achieve the desired system performance objective. Undoubtedly to date we have merely an obscure view of the ultimate applications that the future will bring to the CATV industry, but it is intended that this analysis will provide the foundation necessary to meet the ever increasing challenge that we can expect from the inevitably more sophisticated systems to come.

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APPENDIX 1

Explanation of Notation

Operator Notation

Symbol	Explanation
A/B	A divided by B
A # B, A \ B	COM operators, see Appendix 2

The following notation is used in the text and in the Appendices to represent the system and equipment parameters. Each parameter is represented by 1 or 2 capital letters followed by lower case letters that denote where the parameter is used in the system. The explanation of the letters is provided below the examples.

Example: Nrf = Carrier-to-Noise Ratio of the return feeder system amplifiers.

PFrt = Performance Factor for the total return system.

Capital Letters	Explanation
C	Amplifier cascade.
G	Amplifier gain.
KC	Amplifier cascade per $\sqrt{\text{strand mile}}$
KQ	Amplifier quantity per strand mile (amplifier density).
L	Signal level.
N	Carrier-to-noise ratio, abbreviated as "noise" in the text.
NF	Amplifier noise figure.
OC	Amplifier output capability, the output level of an amplifier at which the carrier-to-cross modulation ratio is 57.
PF	Performance factor = X + 2N
Q	Quantity, the number of amplifiers in a given area.
Sm	The number of strand miles.
X	Carrier-to-cross modulation ratio, abbreviated as "cross-modulation" in the text.
Z12	OC1 - OC2 + G1 - G2 + NF1 - NF2

Lower Case Letters	Explanation
a	Transportation system
f	Feeder system
i	Input
k	Trunk system
o	Output
r	Return system
s	Specification
t	Total
x	Trial value

* *

APPENDIX 2

Fundamental Relationships for COM Operators

Definitions

$$A \ \#p\ B = -10 \log \left[10^{-A/10} + 10^{-B/10}\right] \quad \text{(COMP)}$$

$$A \ p\backslash\ B = -10 \log \left[10^{-A/10} - 10^{-B/10}\right] \quad \text{(DCOMP)}$$

$$A \ \#v\ B = -20 \log \left[10^{-A/20} + 10^{-B/20}\right] \quad \text{(COMV)}$$

$$A \ v\backslash\ B = -20 \log \left[10^{-A/20} - 10^{-B/20}\right] \quad \text{(DCOMV)}$$

Algebraic Rule

If

$C = A \# B$, then $A = C \setminus B$

Distributive Rule

$(A+B) \# (A+C) = A + B \# C$

$(A+B) \setminus (A+C) = A + B \setminus C$

Derivative Rule

$$\frac{d(Y \#v\ C)}{dy} = 10^{-(Y - Y \#v\ C)/20}$$

(where C is a constant)

Associative Rule

$A \# (B \# C) = (A \# B) \# C = (A \# C) \# B$

Inter-Order Conversion

$(2A) \#v\ (2B) = 2(A \#p\ B)$

* *

APPENDIX 3

Maximum Performance

The following is a derivation of the difference in output level required between two systems to obtain the maximum performance.

If we have two sub-systems, 1 and 2, the criteria for setting the levels of system 1 (same criteria for system 2) is:

$$(1) \quad \frac{dXt}{dL1} = 2\,\frac{dNt}{dL1}$$

where $Xt = X1 \#v\ X2$

$Nt = N1 \#p\ N2$

The criteria is illustrated by the following example. Suppose that Xt = 51 and Nt = 43. Suppose also that $\frac{dXt}{dL1}$ is less than $2\,\frac{dNt}{dL1}$, i.e., suppose that increasing the operating level of system 1 by 2 dB results in an increase of Nt by 1 dB and a decrease of Xt by 1 dB. We now decrease all levels by 1 dB; the procedure is summarized in the following table:

	Noise	Cross-Modulation
Initial Performance	43 dB	51 dB
Increase System 1 Levels 2 dB	44	50
Reduce all Levels 1 dB	43	52

Thus, by changing the relationship between the levels of system 1 and system 2 the cross-modulation has been increased by 1 dB with the same noise performance. However, when the difference in operating level meets the maximum performance criteria, the improvement illustrated is not possible. Further reflection will reveal to the reader that the criteria is valid for more than two units operating together, but the two-unit case is sufficient for this paper.

Substituting for Xt and Nt in expression (1) and using the derivative rule for COM operators, we obtain:

$$10^{-(X1 - X1 \#v\ X2)/20} = 2 \times 10^{(N1 - N1 \#p\ N2)/10}$$

$$10^{-(X1 - Xt)/20} = 2 \times 10^{-(N1 - Nt)/10}$$

Taking the LOG of both sides of the equation

$$-(X1 - Xt)/20 = \text{LOG } 2 - (N1 - Nt)/10$$

$$Xt - X1 = 6 - 2(N1 - Nt)$$

by symmetry:

$$Xt - X2 = 6 - 2(N2 - Nt)$$

Solving the first equation for Xt and substituting into the second, we obtain:

$$X1 - 2N1 = X2 - 2N2$$

To find the difference in output level between the two systems we substitute the following expressions for X & N:

$$X = (OG - Lo) \times 2 + 57 - 20 \text{ LOG } C$$

$$N = Li + 59 - NF - 10 \text{ LOG } Q$$

the input level, Li, can be replaced by Lo - G.

Making the above substitutions and solving for L1o - L2o, we obtain:

$$L1o - L2o = (OC1 - OC2 + G1 - G2 + NF1 - NF2)/2 + 10 \text{ LOG } \sqrt{\frac{Q1\ C2}{C1\ Q2}}$$

It is interesting to note that the old idea of running all amplifiers midway between noise and cross-modulation is valid only when Q = C, i.e., for a single cascade of amplifiers.

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APPENDIX 4

Apportionment of Strand Miles Between Hub Systems

Cascade is proportional to $\sqrt{\text{strand miles}}$
Quantity is proportional to strand miles

$$\text{Cascade} = KC \times Sm^{\frac{1}{2}} = C$$

$$\text{Quantity} = KQ \times Sm = Q$$

$$\mathrm{Xrt} = \mathrm{Xr} - 20 \log C = \mathrm{Xr} - 20 \log (\mathrm{KC} \times \mathrm{Sm}^{\frac{1}{2}})$$

$$= \mathrm{Xr} - 20 \log \mathrm{KC} - 20 \log \mathrm{Sm}^{\frac{1}{2}}$$

$$= \mathrm{Xr} - 20 \log \mathrm{KC} - 10 \log \mathrm{Sm}$$

$$\mathrm{Nrt} = \mathrm{Nr} - 10 \log Q = \mathrm{Nr} - 10 \log (\mathrm{KQ} \times \mathrm{Sm})$$

$$= \mathrm{Nr} - 10 \log \mathrm{KQ} - 10 \log \mathrm{Sm}$$

The return system cross-modulation is equal to the combined cross-modulation of the return transportation, trunk and feeder systems.

$$\mathrm{Xrt} = \mathrm{Xra}\ \#v\ \mathrm{Xrk}\ \#v\ \mathrm{Xrf}$$

$$= \left[(\mathrm{OCra} - \mathrm{Lrao}) \times 2 + 57 - 20 \log \mathrm{Cra}\right]$$

$$\#v \left[(\mathrm{OCrk} - \mathrm{Lrko}) \times 2 + 57 - 20 \log \mathrm{Crk}\right]$$

$$\#v \left[(\mathrm{OCrf} - \mathrm{Lrf}) \times 2 + 57 - 20 \log \mathrm{Crf}\right]$$

Substitute for Lrko and Lrfo using formula for maximum performance from Appendix 3, i.e.:

$$\mathrm{Lrko} = \mathrm{Lrao} + (\mathrm{OCrk} - \mathrm{OCra} + \mathrm{Grk} - \mathrm{Gra} + \mathrm{NFrk} - \mathrm{NFra})/2 + 10 \log \sqrt{\frac{\mathrm{Qrk}}{\mathrm{Crk}}}$$

$$10 \log \sqrt{\frac{\mathrm{Qrk}}{\mathrm{Crk}}} = 10 \log \sqrt{\frac{\mathrm{KQrk}\ \mathrm{Sm}}{\mathrm{KCrk}\ \mathrm{Sm}^{\frac{1}{2}}}} =$$

$$= 10 \log \sqrt{\frac{\mathrm{KQrk}}{\mathrm{KCrk}}} + 10 \log \sqrt{\mathrm{Sm}^{\frac{1}{2}}}$$

Let Zrka = OCrk - OCra + Grk - Gra + NFrk - NFra

$$\mathrm{Xrt} = \left[2 \times \mathrm{OCra} - 2 \times \mathrm{Lrao} + 57 - 20 \log \mathrm{Cra}\right]$$

$$\#v \left[2 \times \mathrm{OCrk} - 2 \times \mathrm{Lrao} - \mathrm{Zrka} - 10 \log\left(\frac{\mathrm{KQrk}}{\mathrm{KCrk}}\right) - 10 \log \mathrm{Sm}^{\frac{1}{2}} + 57 - 20 \log(\mathrm{KCrk} \times \mathrm{Sm}^{\frac{1}{2}})\right]$$

$$\#v \left[2 \times \mathrm{OCrf} - 2 \times \mathrm{Lrao} - \mathrm{Zrfa} - 10 \log\left(\frac{\mathrm{KQrf}}{\mathrm{Crf}}\right) - 10 \log \mathrm{Sm} + 57 - 20 \log \mathrm{Crf}\right]$$

$$\mathrm{Xrt} = \left[2 \times \mathrm{OCra} - 2 \times \mathrm{Lrao} + 57 - 20 \log \mathrm{Cra}\right]$$

$$\#v \left[2 \times \mathrm{OCrk} - 2 \times \mathrm{Lrao} + 57 - 10 \log \mathrm{Sm}^{3/2} - \mathrm{Zrka} - 10 \log (\mathrm{KQrk} \times \mathrm{KCrk})\right]$$

$$\#v \left[2 \times \mathrm{OCrf} - 2 \times \mathrm{Lrao} + 57 - 10 \log \mathrm{Sm} - \mathrm{Zrfa} - 10 \log (\mathrm{KQrf} \times \mathrm{Crf})\right]$$

$$\mathrm{Xrt} = 57 - 2 \times \mathrm{Lrao} + \left[2 \times \mathrm{OCra} - 20 \log \mathrm{Cra}\right]$$

$$\#v \left[2 \times \mathrm{OCrk} - 10 \log \mathrm{Sm}^{3/2} - \mathrm{Zrka} - 10 \log \mathrm{KQrk} \times \mathrm{KCrk}\right]$$

$$\#v \left[2 \times \mathrm{OCrf} - 10 \log \mathrm{Sm} - \mathrm{Zrfa} - 10 \log \mathrm{KQrf} \times \mathrm{Crf}\right]$$

The return system noise is equal to the combined noise of the return transportation, trunk and feeder systems.

$$\mathrm{Nrt} = \mathrm{Nra}\ \#p\ \mathrm{Nrk}\ \#p\ \mathrm{Nrf}$$

$$= (\mathrm{Lrao} - \mathrm{Gra} + 59 - \mathrm{NFra} - 10 \log \mathrm{Cra})$$

$$\#p\ (\mathrm{Lrko} - \mathrm{Grk} + 59 - \mathrm{NFrk} - 10 \log \mathrm{Qrk})$$

$$\#p\ (\mathrm{Lrfo} - \mathrm{Grf} + 59 - \mathrm{NFrf} - 10 \log \mathrm{Qrf})$$

Substituting for Lrko, Lrf, Qrk and Qrf

$$\mathrm{Nrt} = (\mathrm{Lrao} - \mathrm{Gra} + 59 - \mathrm{NFra} - 10 \log \mathrm{Cra})$$

$$\#p\ \left(\mathrm{Lrao} + \mathrm{Zrka}/2 + 10 \log \sqrt{\frac{\mathrm{KQrk}}{\mathrm{KCrk}}} + 10 \log \sqrt{\mathrm{Sm}^{\frac{1}{2}}} - \mathrm{Grk} + 59 - \mathrm{NFrk} - 10 \log \mathrm{KQrk} - 10 \log \mathrm{Sm}\right)$$

$$\#p\ \left(\mathrm{Lrao} + \mathrm{Zrfa}/2 + 10 \log \sqrt{\frac{\mathrm{KQrf}}{\mathrm{Crf}}} + 10 \log \sqrt{\mathrm{Sm}} - \mathrm{Grf} + 59 - \mathrm{NFrf} - 10 \log \mathrm{KQrf} - 10 \log \mathrm{Sm}\right)$$

$$\mathrm{Nrt} = (\mathrm{Lrao} - \mathrm{Gra} + 59 - \mathrm{NFra} - 10 \log \mathrm{Cra})$$

$$\#p\ \left(\mathrm{Lrao} + \mathrm{Zrka}/2 - 10 \log \sqrt{\mathrm{KCrk} \times \mathrm{KQrk}} - 10 \log \sqrt{\mathrm{Sm}^{3/2}} - \mathrm{Grk} + 59 - \mathrm{NFrk}\right)$$

$$\#p\ \left(\mathrm{Lrao} + \mathrm{Zrfa}/2 - 10 \log \sqrt{\mathrm{Crf} \times \mathrm{KQrf}} - 10 \log \sqrt{\mathrm{Sm}} - \mathrm{Grf} + 59 - \mathrm{NFrf}\right)$$

$$\mathrm{Nrt} = \mathrm{Lrao} + 59 + (-\mathrm{Gra} - \mathrm{NFra} - 10 \log \mathrm{Cra})$$

$$\#p\ \left(\mathrm{Zrka}/2 - 10 \log \sqrt{\mathrm{KCrk} \times \mathrm{KQrk}} - 10 \log \sqrt{\mathrm{Sm}^{3/2}} - \mathrm{Grk} - \mathrm{NFrk}\right)$$

$$\#p\ \left(\mathrm{Zrfa}/2 - 10 \log \sqrt{\mathrm{Crf} \times \mathrm{KQrf}} - 10 \log \sqrt{\mathrm{Sm}} - \mathrm{Grf} - \mathrm{NFrf}\right)$$

The return system performance factor, PFrt = Xrt + 2Nrt

$$= 57 + 2 \times 59 + \left[2 \times \mathrm{OCra} - 20 \log \mathrm{Cra}\right]$$

$$\#v \left[2 \times \mathrm{OCrk} - 10 \log \mathrm{Sm}^{3/2} - \mathrm{Zrka} - 10 \log (\mathrm{KQrk} \times \mathrm{KCrk})\right]$$

$$\#v \left[2 \times \mathrm{OCrf} - 10 \log \mathrm{Sm} - \mathrm{Zrfa} - 10 \log (\mathrm{KQrf} \times \mathrm{Crf})\right]$$

$$+ \left[-2\mathrm{Gra} - 2\mathrm{NFra} - 20 \log \mathrm{Cra}\right]$$

$$\#p \left[-2\mathrm{Grk} - 2\mathrm{NFrk} - 10 \log \mathrm{Sm}^{3/2} + \mathrm{Zrka} - 10 \log (\mathrm{KCrk} \times \mathrm{KQrk})\right]$$

$$\#p \left[-2\mathrm{Grf} - 2\mathrm{NFrf} - 10 \log \mathrm{Sm} + \mathrm{Zrfa} - 10 \log (\mathrm{Crf} \times \mathrm{KQrf})\right]$$

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APPENDIX 5

Meeting Performance with Maximum Feeder Levels

Let Nt and Xt be the combined noise and the combined cross-modulation of the elements of the system other than the amplifiers in the forward feeder. Let Ntx and Xtx be trial values of the above that are calculated at some arbitrary operating level of the system. The utility of these latter values will be seen below.

Since the noise of a system increases 1 dB for each 1 dB increase of operating level and the cross-modulation decreases 2 dB for each 1 dB increase in operating level, it follows that:

$$\mathrm{Nt} = \mathrm{Ntx} + \Delta \mathrm{L}$$

$$\mathrm{Xt} = \mathrm{Xtx} - 2\Delta \mathrm{L}$$

where ΔL is the change in operating level.

If we multiply the first expression by 2 and add to the second expression we obtain:

$$\mathrm{Xt} + 2\,\mathrm{Nt} = \mathrm{Xtx} + 2\mathrm{Ntx} \tag{1}$$

The quantity X + 2N is independent of level for any system and is herein defined as the Performance Factor, PF.

If we let Ns and Xs be the system noise and cross-modulation specifications and let Nf and Xf be the feeder noise and cross-modulation that we are attempting to calculate, it is true that:

$$\mathrm{Nt}\ \#p\ \mathrm{Nf} = \mathrm{Ns}$$

$$\mathrm{Xt}\ \#v\ \mathrm{Xf} = \mathrm{Xs}$$

by definition:

$$\mathrm{Nt} = \mathrm{Ns}\ p\backslash\ \mathrm{Nf}$$

$$\mathrm{Xt} = \mathrm{Xs}\ v\backslash\ \mathrm{Xf}$$

substituting into expression (1):

$$\mathrm{Xs}\ v\backslash\ \mathrm{Xf} + 2(\mathrm{Ns}\ p\backslash\ \mathrm{Nf}) = \mathrm{Xtx} + 2\mathrm{Ntx}$$

$$\mathrm{Xs}\ v\backslash\ \mathrm{Xf} = \mathrm{Xtx} + 2\,\mathrm{Ntx} - 2(\mathrm{Ns}\ p\backslash\ \mathrm{Nf})$$

$$\mathrm{Xs} = \left[\mathrm{Xtx} + 2\mathrm{Ntx} - 2(\mathrm{Ns}\ p\backslash\ \mathrm{Nf})\right]\ \#v\ \mathrm{Xf}$$

$$\mathrm{Xf} = \mathrm{Xs}\ v\backslash\left[\mathrm{Xtx} + 2\mathrm{Ntx} - 2(\mathrm{Ns}\ p\backslash\ \mathrm{Nf})\right] \tag{2}$$

Xtx + 2Ntx is the performance factor, PFtx, of the non-feeder system and can be calculated.

Nf can be found using expression (1)

$$\mathrm{Nf} = (\mathrm{Xfx} + 2\mathrm{Nfx} - \mathrm{Xf})/2$$

substituting for Nf in expression (2):

$$\mathrm{Xf} = \mathrm{Xs}\ v\backslash\left[\mathrm{PFt} - 2(\mathrm{Ns}\ p\backslash\left[\mathrm{Xfx} + 2\mathrm{Nfx} - \mathrm{Xf}\right]/2)\right]$$

Applying the COM operator rule for inter-order conversion to the last terms and substituting for Xtx + 2Ntx:

$$\mathrm{Xf} = \mathrm{Xs}\ v\backslash\left[\mathrm{PFt} - (2\mathrm{Ns}\ v\backslash\left[\mathrm{PFf} - \mathrm{Xf}\right]/2)\right]$$