MODULATION INDEX AND TRANSMITTER POWER RELATIONSHIP IN MULTIPLE CHANNEL FM SYSTEMS

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In a recent paper titled "Understanding Laser Link's FDM/FM Airlink System", appearing in this month's issue of Cable News (copies available on request from Laser Link Corp.), the authors reviewed the operation of frequency division multiplexed FM systems. A point was made of the fact that the FM system superposes the various subcarriers and behaves as each one existed alone in its part of the output spectrum. This corresponds to the experience of many engineers who have tested a visual carrier for SNR and an aural carrier for SNR on a single TV channel FM microwave system and have observed that the signal to noise ratios of the individual carriers are independent of the presence of absence of the other subcarriers. By the same process of superposition, it is possible to determine the effective spread of energy from the carrier to the sidebands when many television channels are combined by frequency division multiplex (see figures 1 and 2) and then impressed as a modulation drive on an FM system. In all FM systems, the concept of energy left in the carrier as opposed to energy in the sidebands must be dealt with using one precaution. Recalling that an AM system, under conditions of increasing modulation, shows increasing energy delivered to the sidebands, until 100% of modulation is reached, the communications engineer is careful to avoid further drive because this will result in overmodulation and distortion of information. 100% modulation in AM corresponds to 50% of energy in the carrier, and 50% of energy in the sidebands. If one sideband is fully suppressed, 66% of the energy is left in the carrier, and 33% is in the unsuppressed sideband. As a practical matter, suppressed sideband television with average picture content, and with the FCC standard for 0.75 mhz of the suppressed sideband still carried on the air, 46% (sideband energy)- 54% (carrier energy) relationship is typical. With FM, there is no corresponding physical limitation on the depth of modulation, other than the quality of the FM equipment, and the limitations of spectrums spread imposed by legal use of bandwidth. Thus with FM systems, equal effective use of power as compared with a 100% modulated AM carrier corresponds to a modulation index of approximately 1.00 (see references 1 and 2). However, the FM system can be driven harder and harder into modulation, and at modulation index of 2.403, 100% of the energy is then in the sidebands. Further drive is quite practical with a linear FM system, producing well known the condition of improvements of signal to noise ratio with greater modulation indexes. Under these conditions, the ratio of energy in sidebands to energy in carrier grows poorer, that is to say the carrier then shows an increasing percentage of power at the expense of energy in the sidebands, while signal to noise ratio is increasing. Therefore to use the analogy of energy in sidebands as an indicator of useful distribution of energy, one must confine the area of interest to values of modulation index below 2.403. Above that value of modulation index, the analogy can be extended by using the effective power in sidebands, and referring to them as more than 100%. Thus with a modulation index 2.93 one can refer to the energy in the sidebands as being effectively 150%.

As indicated above, the multichannel FM system can be analyzed as if each subcarrier (channel) existed separately. Therefore one can take each information channel (TV channel in our case) and determine how much energy exists in the sideband as a result of its individual depth of modulation. Typical spectrum analysis photos of spectrum content of Laser Link's multi-channel FM transmitter are shown in figures 3-6. In the airlink system, no individual channel has a modulation index that exceeds 2.403 so that the energy in the sidebands due to that channel alone can be determined by reference to a table of bessel function that shows the drop in carrier power at each channel modulation index. The remaining power is in the sidebands. Then by adding the power in the sidebands due to each of the individual channels, the effective total of energy in the sidebands can be found. Where this energy is less than 100%, this is the total sideband energy, and an effective overall modulation index corresponding to this can be computed. Where the effective sideband energy is more than 100%, then the effective overall modulation index can be computed corresponding to modulation of more than 2.403.

The above computations have been performed for the Airlink System in tables 1 thru 11, for 5 thru 18 channels of television on a single Airlink system. The results of these tables have been summarized on table 12. This shows that with the poorest modulation indexes used (limitations of bandwidth under F. C. C. Rules and Regulations), the modulation index is 0.96 corresponding to the same effective use of power as an AM system. However, being a single channel FM system, as much as 60 watts of power is available for use at a single transmitter location. In the case of higher modulation indexes, as much as 3.59 improvement is available over the corresponding AM system on the basis of watt for watt equivalence. This is more than 10 db of improvement of a watt for watt equivalence for AM.

Another method for determining the effective power of an FDM/FM transmitter in terms of modulation index is to set up an equivalence relationship between power required for individual AM channels to produce the same SNR as the channels of an FDM/FM system. By adding all the individual AM powers required, one gets the total AM transmitter power needed for equivalence. The FM channels all use the same transmitter power, as indicated above, so that a direct comparison is possible, on the basis of identical paths, identical antenna gains and identical waveguide sections. The derivation that is given below uses the terms "A", "B", "C", and "D" to represent air path, antenna and waveguide. Since these are indentical for AM and FM systems, they cancel in the comparison, making the relationship general for all cases.

Ref. 1 ITT Radio Engineers Handbook 21-12.

Ref. 2 Mischa Schwartz, Information Transmission, Modulation and Noise, page 305, McGraw Hill and Company, 1959.

AM System .X milliwatts=x dBm for an individual TV channel delivered to an antenna feed.
.5MHZ bandwidth/Channel
.Noise figure 12dB
.Path loss "A" dB
.Transmitter antenna gain "B" dB
.Receiver antenna gain "C" dB
.Incidental waveguide and fitting losses "D" dB
.Modulation 100%

Carrier power at receiver=x-A-D+B+C dBm
Receiver channel noise power in 5MHZ=10 $\mathrm{Log}_{10} \frac{5}{1} = 7\mathrm{dB}$

Noise power of receiver= -114+12+7= -95dBm
Channel carrier to noise ratio=x-A-D+B+C- (-95)
Channel signal to noise ratio= 95+x-A-D+B+C

If AM and FM system have same SNR

$95+x-A-D+B+C=136-A-D+B+C+p$
$95 + x-136=P$
$x-41=p=10\ \mathrm{Log}_{10}(M)^2 = 10\ \mathrm{Log}_{10} X-41$

$-41= 10\ \mathrm{Log}_{10} \frac{1}{12600}$ $\qquad X=12600\ M^2$

This is a relationship between power per AM channel to channel modulation. Index for equal signal to noise ratio.

The relationship $X=12600M^2$ is plotted as a curve in Figure 7.

The comparison for airlink systems of 5 to 18 channels, operating at the modulation indexes of tables 1 thru 11 is given as table 13. As can be seen, the equivalent AM system would require transmitted power of 10.38 to 89.00 watts.

Equivalent AM Power Computation

FM System Conditions .20 watts of power=+43 dBm
.250 MHZ bandwidth
.Noise figure 12dB
.Path loss "A" dB
.Transmitter antenna gain "B" dB
.Receiver antenna gain "C" dB
.Incidental waveguide and fitting losses "D" dB
.Channel modulation Index=M

Computation:

Carrier power at Receiver= 43-A-D+B+C dBm

Noise power in 250 MHZ=10 $\text{Log}_{10}\frac{250}{1}$ = 24 dB above 1 MHZ

Noise power of receiver= -114+12+24= -78 dBm

Receiver carrier to noise ratio= 43-A-D+B+C- (-78)
= 43+78-A-D+B+C
= 121-A-D+B+C

Video bandwidth per TV channel (2 sidebands)=8MHZ

Noise bandwidth reduction= 10 $\text{Log}_{10}\frac{8}{250}$ =+15dB

Modulation Index (referred to 1.00) as
a factor in SNR= 10 $\text{Log}_{10}\left(\frac{M}{1}\right)^2$ = p (dB)

Channel signal to noise ratio= 121-A-D+B+C+15+p
= 136-A-D+B+C + p

This presumes a receiver input above the FM threshold of 10 dB above receiver noise figure. In actual practice SNRS are designed to stay above 35dB in worst case conditions, for Airlink systems.

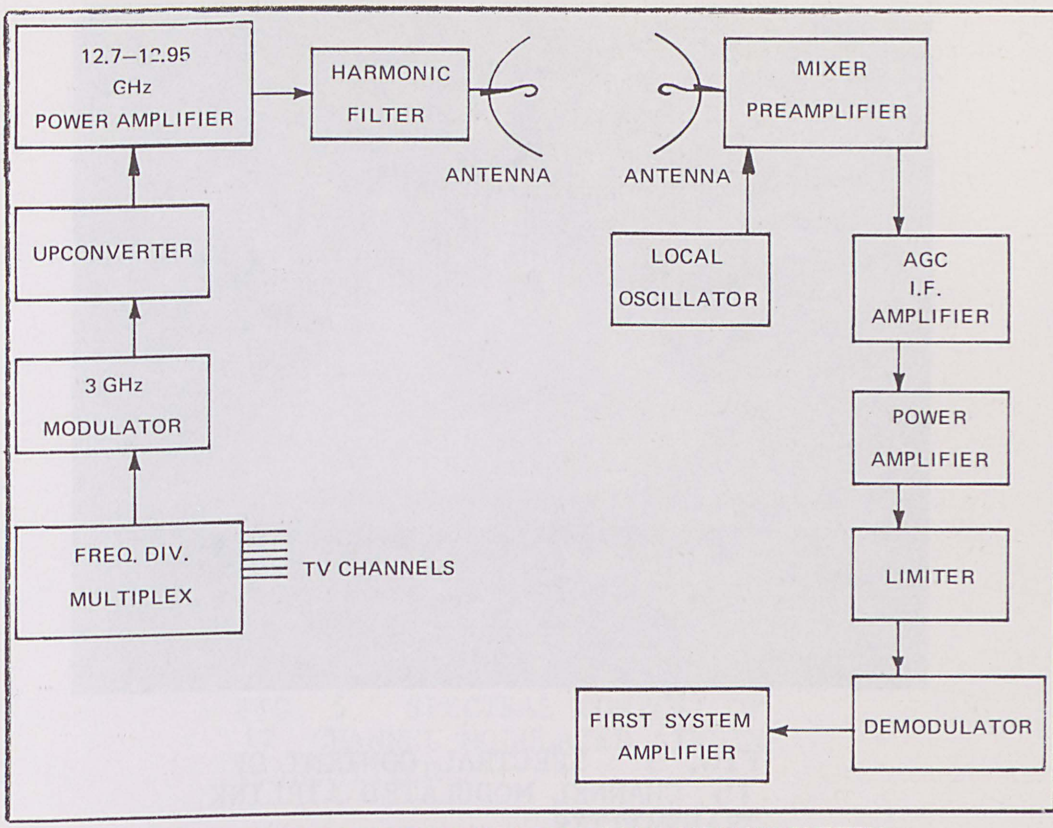

FIG. 1 BLOCK DIAGRAM OF LASER LINK AIRLINK SYSTEM

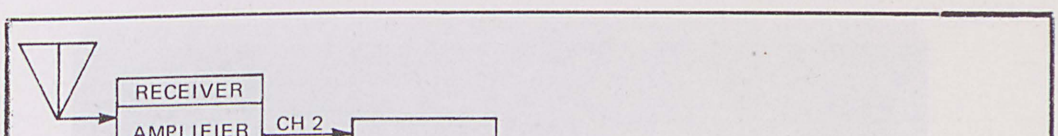

FIG. 2 FREQUENCY DIVISION MULTIPLEXING OFF THE AIR

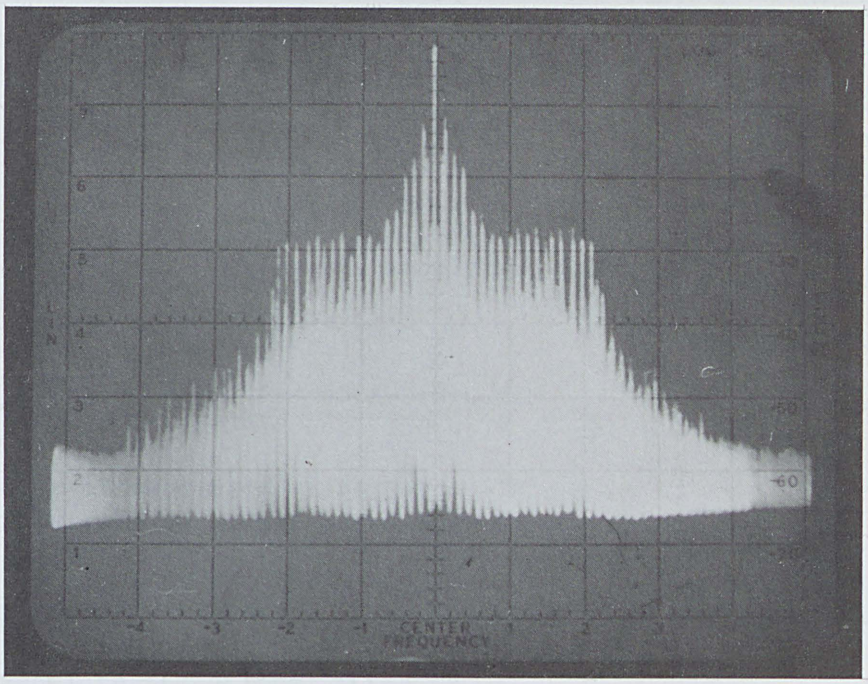

FIG. 3 SPECTRAL CONTENT OF 15 CHANNEL MODULATED AIRLINK TRANSMITTER

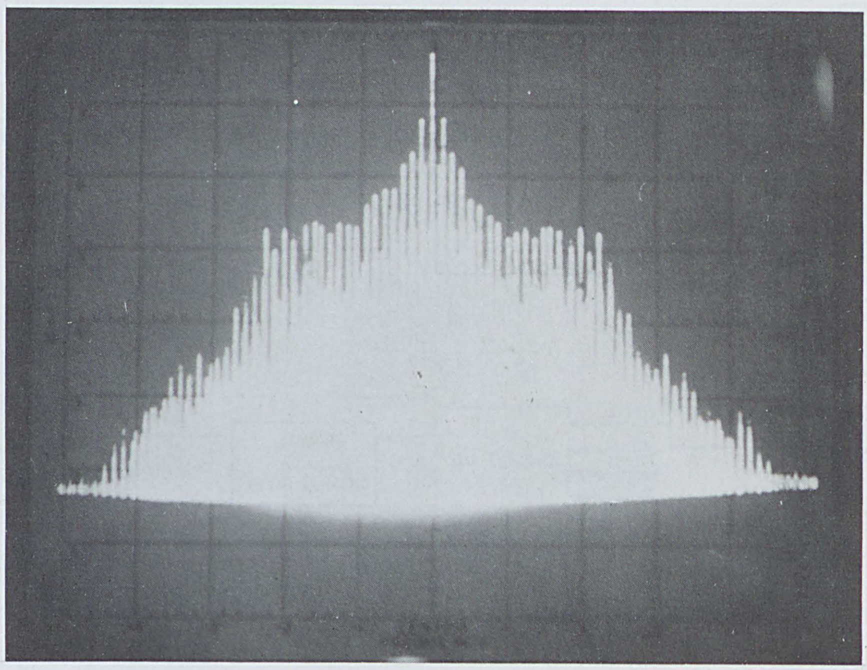

FIG. 4 SPECTRAL CONTENT OF 16 CHANNEL MODULATED AIRLINK TRANSMITTER

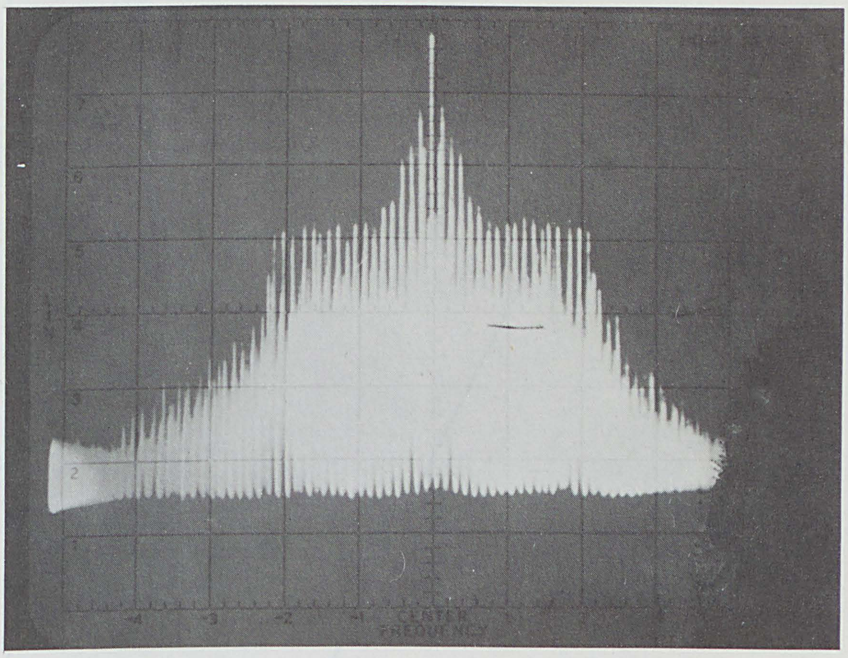

FIG. 5 SPECTRAL CONTENT OF 17 CHANNEL MODULATED AIRLINK TRANSMITTER

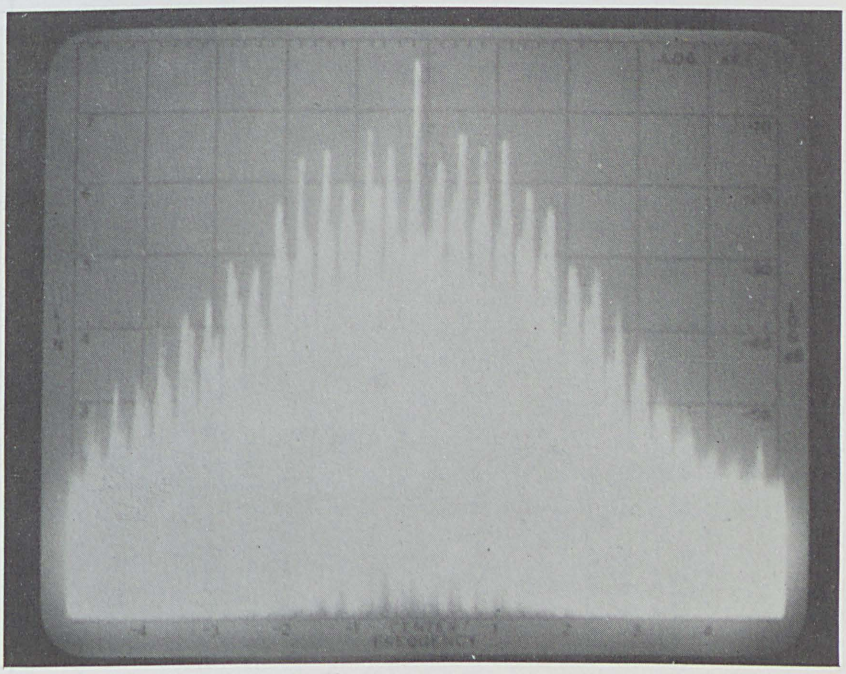

FIG. 6 SPECTRAL CONTENT OF 6 CHANNEL MODULATED AIRLINK TRANSMITTER

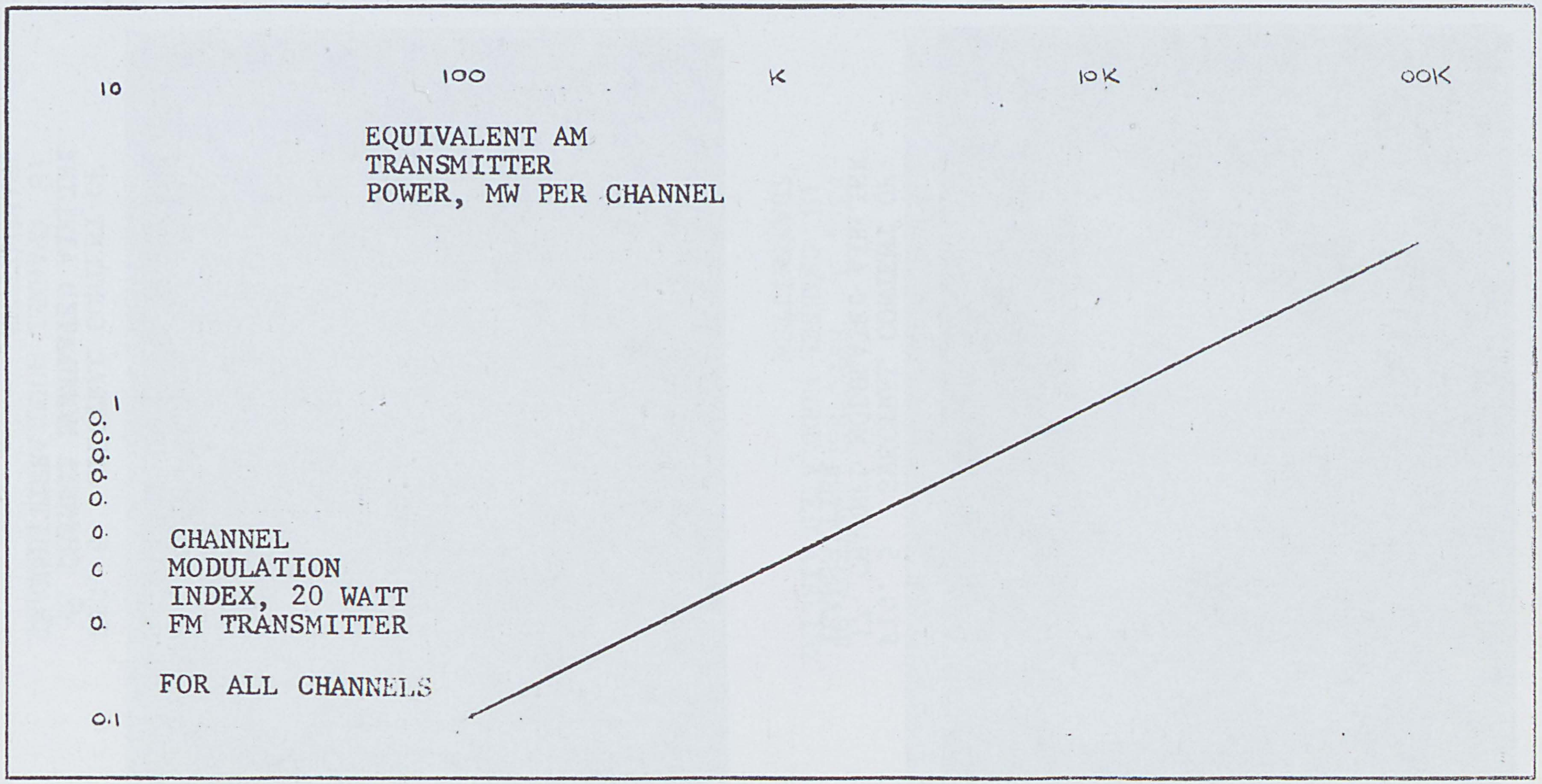

FIGURE 7. PLOT OF CHANNEL MODULATION INDEX M AGAINST THE EQUIVALENT AM TRANSMITTER POWER X TO PRODUCE THE SAME SIGNAL TO NOISE RATIO.

TABLE 1

NUMBER OF CHANNELS: 5

Channel Desig.	Freq. of Video Carr. MHz	Channel Mod. Index	FM Carrier Level for this Channel alone is at its Mod. Index	Power in Carrier	Equivalent Power in Sidebands
T 7	7.	1.00	.765	.585	.415
T 8	13.	1.00	.765	.585	.415
T 9	19.	0.97	.878	.770	.230
T 10	25.	0.96	.883	.780	.220
T 11	31.	0.96	.883	.780	.220
					1.500

This corresponds to 150% of energy in sidebands. In single channel FM, 100% of the energy is in the sidebands at a modulation index equal to 2.403. The equivalent of 150% of power in sidebands occurs at a modulation index of 2.403 x $\sqrt{1.5}$ = 2.403 x 1.225 = 2.93.

TABLE 2

NUMBER OF CHANNELS: 6

Channel Desig.	Freq. of Video Carr. MHz	Channel Mod. Index	FM Carrier Level for this Channel alone is at its Mod. Index	Power in Carrier	Equivalent Power in Sidebands
T 7	7	0.86	.824	.680	.320
T 8	13	0.77	.857	.735	.265
T 9	19	0.66	.894	.800	.200
T 10	25	0.62	.906	.822	.178
T 11	31	0.62	.906	.822	.178
T 12	37	0.60	.912	.832	.168
					1.309

This corresponds to 131% of energy in sidebands. In a single channel FM, 100% of the energy is in the sidebands at a modulation index of 2.403. The equivalent of 131% of power in sidebands occurs at a modulation index of $2.403 \times \sqrt{1.309} =$ $2.4 \times 1.14 = 2.73$

TABLE 3

NUMBER OF CHANNELS: 7

Channel Desig.	Freq. of Video Carr. MHz	Channel Mod. Index	FM Carrier Level for this Channel alone is at its Mod. Index	Power in Carrier	Equivalent Power in Sidebands
T 7	7	0.72	.874	.746	.254
T 8	13	0.50	.939	.873	.127
T 9	19	0.45	.950	.903	.097
T 10	26	0.40	.960	.922	.078
T 11	31	0.37	.966	.933	.067
T 12	37	0.36	.968	.937	.063
T 13	43	0.35	.970	.941	.059
					.645

64.5% of energy is in the sidebands. This is equivalent to an overall carrier level of $\sqrt{1.645} = \sqrt{.355} = .595$ at a modulation index equal to 1.34.

TABLE 4

NUMBER OF CHANNELS: 8

Channel Desig.	Freq. of Video Carr. MHz	Channel Mod. Index	FM Carrier Level for this Channel alone is at its Mod. Index	Power in Carrier	Equivalent Power in Sidebands
T 7	7	0.67	.891	.794	.216
T 8	13	0.40	.960	.922	.078
T 9	19	0.30	.978	.956	.044
T 10	25	0.25	.984	.968	.032
T 11	31	0.23	.987	.974	.026
T 12	37	0.21	.989	.978	.022
T 13	43	0.20	.990	.980	.020
S 1	49	0.19	.991	.982	.018
					.456

45.6% of the energy is in the sidebands. This is equivalent to an overall carrier level of:
$\sqrt{1.456} = \sqrt{.544} = .737$ at a modulation index of 1.06.

TABLE 5

NUMBER OF CHANNELS: 12*

Channel Desig.	Freq. of Video Carr. MHz	Channel Mod. Index	FM Carrier Level for this Channel alone is at its Mod. Index	Power in Carrier	Equivalent Power in Sidebands
T 7	7	1.70	.398	.159	.841
T 8	13	0.92	.799	.639	.361
T 9	19	0.63	.903	.815	.185
T 10	25	0.48	.943	.889	.111
T 11	31	0.38	.962	.926	.074
T 12	37	0.32	.975	.950	.050
T 13	43	0.28	.981	.962	.038
2	55	0.26	.983	.966	.034
3	61	0.26	.983	.966	.034
4	67	0.26	.983	.966	.034
5	77	0.26	.983	.966	.034
6	83	0.26	.983	.966	.034
					1.830

This corresponds to 183% of energy in sidebands. In a single channel FM, 100% of the energy is in the sidebands at a modulation index of 2.403. The equivalent of 183% of power in sidebands occurs at a modulation index of $2.403 \sqrt{1.83} = 2.403 \times 1.352 = 3.26$.

* 9,10 and 11 channels are derived by deletion of channels of the 12-channel baseband while remaining channels operated at the tabulated values.

TABLE 6

NUMBER OF CHANNELS: 13

Channel Desig.	Freq. of Video Carr. MHz	Channel Mod. Index	FM Carrier Level for this Channel alone is at its Mod. Index	Power in Carrier	Equivalent Power in Sidebands
T 7	7	2.02	.212	.045	.955
T 8	13	1.08	.729	.530	.470
T 9	19	0.74	.868	.753	.247
T 10	25	0.57	.920	.847	.153
T 11	31	0.45	.950	.903	.097
T 12	37	0.38	.964	.929	.071
T 13	43	0.33	.973	.947	.053
S 1	49	0.30	.978	.956	.046
2	55	0.30	.978	.956	.046
3	61	0.30	.978	.956	.046
4	67	0.30	.978	.956	.046
A	73	0.30	.978	.956	.046
S 5	79	0.30	.978	.956	.046
					2.322

This corresponds to 232% of energy in sidebands. In a single channel FM, 100% of the energy is in the sidebands at a modulation index of 2.403. The equivalent of 232% of power in sidebands occurs at a modulation index of 2.430 x $\sqrt{2.322}$ = 2.403 x 1.523 = 3.59.

TABLE 7

NUMBER OF CHANNELS: 14

Channel Desig.	Freq. of Video Carr. MHz	Channel Mod. Index	FM Carrier Level for this Channel alone is at its Mod. Index	Power in Carrier	Equivalent Power in Sidebands
T 7	7	1.54	.589	.346	.654
T 8	13	0.83	.835	.697	.303
T 9	19	0.57	.920	.847	.153
T 10	25	0.43	.954	.911	.089
T 11	31	0.35	.970	.941	.059
T 12	37	0.29	.979	.958	.042
T 13	43	0.25	.984	.968	.032
S 1	49	0.23	.987	.974	.026
2	55	0.23	.987	.974	.026
3	61	0.23	.987	.974	.026
4	67	0.23	.987	.974	.026
A	73	0.23	.987	.974	.026
S 5	79	0.23	.987	.974	.026
S 6	85	0.23	.987	.974	.026
					1.514

This corresponds to 151% of energy in sidebands. In a single channel FM, 100% of the energy is in the sidebands at a modulation index of 2.403. The equivalent of 151% of power in the sidebands occurs at a modulation index of $\sqrt{2.403}$ 1.51 = 2.403 x 1.23 = 2.96.

TABLE 8

NUMBER OF CHANNELS: 15

Channel Desig.	Freq. of Video Carr. MHz	Channel Mod. Index	FM Carrier Level for this Channel alone is at its Mod. Index	Power in Carrier	Equivalent Power in Sidebands
T 7	7	1.22	.661	.437	.563
T 8	13	0.66	.894	.800	.200
T 9	19	0.45	.950	.902	.098
T 10	25	0.34	.971	.943	.057
T 11	31	0.28	.981	.962	.038
T 12	37	0.23	.987	.974	.026
T 13	43	0.20	.990	.980	.020
S 1	49	0.18	.992	.984	.016
2	55	0.18	.992	.984	.016
3	61	0.18	.992	.984	.016
4	67	0.18	.992	.984	.016
A	73	0.18	.992	.984	.016
S 5	79	0.18	.992	.984	.016
S 6	85	0.18	.992	.984	.016
B	91	0.18	.992	.984	.016
					1.130

This corresponds to 113% of energy in sidebands. In a single channel FM, 100% of the energy is in the sidebands at a modulation index of 2.403. The equivalent of 113% of power in the sidebands occurs at a modulation index of $2.403\sqrt{1.13} = 2.403 \times 1.063 = 2.57$.

TABLE 9

NUMBER OF CHANNELS: 16

Channel Desig.	Freq. of Video Carr. MHz	Channel Mod. Index	FM Carrier Level for this Channel alone is at its Mod. Index	Power in Carrier	Equivalent Power in Sidebands
T 7	7	0.97	.778	.606	.394
T 8	13	0.52	.934	.872	.128
T 9	19	0.27	.982	.964	.036
T 10	25	0.22	.988	.976	.024
T 11	31	0.18	.992	.984	.016
T 12	37	0.16	.993	.986	.014
T 13	43	0.15	.994	.988	.012
S 1	49	0.15	.994	.988	.012
2	55	0.15	.994	.988	.012
3	61	0.15	.994	.988	.012
4	67	0.15	.994	.988	.012
A	73	0.15	.994	.988	.012
S 5	79	0.15	.994	.988	.012
S 6	85	0.15	.994	.988	.012
B	91	0.15	.994	.988	.012
C	97	0.15	.994	.988	.012
					.732

73.2% of energy is in the sidebands. This is equivalent to an overall carrier level of $\sqrt{1.732} = \sqrt{.268} = .516$, which corresponds to a modulation index of 1.49.

TABLE 10

NUMBER OF CHANNELS: 17

Channel Desig.	Freq. of Video Carr. MHz	Channel Mod. Index	FM Carrier Level for this Channel alone is at its Mod. Index	Power in Carrier	Equivalent Power in Sidebands
T 7	7	0.76	.861	.741	.259
T 8	13	0.41	.958	.918	.092
T 9	19	0.28	.981	.962	.038
T 10	25	0.21	.989	.978	.022
T 11	31	0.17	.993	.986	.014
T 12	37	0.14	.995	.990	.010
T 13	43	0.12	.996	.992	.008
S 1	49	0.11	.997	.994	.006
2	55	0.11	.997	.994	.006
3	61	0.11	.997	.994	.006
4	67	0.11	.997	.994	.006
A	73	0.11	.997	.994	.006
S 5	79	0.11	.997	.994	.006
S 6	85	0.11	.997	.994	.006
B	91	0.11	.997	.994	.006
C	97	0.11	.997	.994	.006
D	103	0.11	.997	.994	.006
					.503

50.3% of energy is in the sidebands. This is equivalent to an overall carrier level of 1.503 = .497 = .705, which corresponds to a modulation index of 1.13.

Figure 25. 32 Amplifier Cascade

TABLE 11

NUMBER OF CHANNELS: 18

Channel Desig.	Freq. of Video Carr. MHz	Channel Mod. Index	FM Carrier Level for this Channel alone is at its Mod. Index	Power in Carrier	Equivalent Power in Sidebands
T 7	7	0.66	.894	.800	.200
T 8	13	0.36	.968	.937	.063
T 9	19	0.25	.984	.968	.032
T 10	25	0.19	.991	.982	.018
T 11	31	0.15	.994	.988	.012
T 12	37	0.13	.996	.992	.008
T 13	43	0.11	.997	.994	.006
S 1	49	0.10	.998	.996	.004
2	55	0.10	.998	.996	.004
3	61	0.10	.998	.996	.004
4	67	0.10	.998	.996	.004
A	73	0.10	.998	.996	.004
S 5	79	0.10	.998	.996	.004
S 6	85	0.10	.998	.996	.004
B	91	0.10	.998	.996	.004
C	97	0.10	.998	.996	.004
D	103	0.10	.998	.996	.004
E	109	0.10	.998	.996	.004
					.383

38.3% of energy is in the sidebands. This is equivalent to an overall carrier level of $\sqrt{1.383} = \sqrt{.617} = .785$, which corresponds to a modulation index of 0.95.

TABLE 12

TABULATION OF POWER IN SIDEBAND COMPUTATION

Number of Channels	Equivalent % Power in Sidebands (based on 100% of sideband power at a Mod. index of 2.403).	Corresponding Index of Modulation for Multi-Channel System.
5	150%	2.93
6	131%	2.73
7	65%	1.34
8	46%	1.06
12 *	183%	3.26
13	232%	3.59
14	151%	2.96
15	113%	2.57
16	73%	1.49
17	50%	1.13
18	38%	0.95

* (9, 10, 11)

Sheet 1 of 2 TABLE 13

EQUIVALENT POWER IN MILLIWATTS REQUIRED TO PRODUCE EQUAL SIGNAL TO NOISE RATIO AS 20 WATT AIRLINK FDM/FM SYSTEM

Number of Channels:	5	6	7	8	12	13
Channel Desig.						
T 7	12,600	9,500	6,600	5,800	40,000	51,000
T 8	12,600	7,600	3,400	2,000	11,000	15,000
T 9	12,000	5,500	2,600	1,200	5,000	7,000
T 10	11,600	4,800	2,000	800	3,000	3,600
T 11	11,600	4,800	1,750	660	1,800	2,600
T 12	N/A	4,600	1,600	550	1,300	1,800
T 13	-	-	1,550	500	1,000	1,400
S 1	-	-	-	450	-	1,100
2	-	-	-	-	850	1,100
3	-	-	-	-	850	1,100
4	-	-	-	-	850	1,100
A	-	-	-	-	-	1,100
5 or S5	-	-	-	-	850	1,100
6 or S6	-	-	-	-	850	-
B	-	-	-	-	-	-
C	-	-	-	-	-	-
D	-	-	-	-	-	-
E	-	-	-	-	-	-
TOTAL	60,400	36,800	19,500	11,960	67,350	89,000

TOTAL REQUIRED EQUIVALENT AM POWER FOR EQUAL S/N.

Sheet 2 of 2

TABLE 13

EQUIVALENT POWER IN MILLIWATTS REQUIRED TO PRODUCE EQUAL SIGNAL TO NOISE RATIO AS 20 WATT AIRLINK FDM/FM SYSTEM

Number of Channels:	14	15	16	17	18
Channel Desig.					
T 7	31,000	20,000	12,000	7,600	5,500
T 8	9,000	5,500	3,500	2,200	1,600
T 9	4,200	2,600	900	1,000	800
T 10	2,300	1,500	600	550	450
T 11	1,500	1,000	400	360	280
T 12	1,100	660	320	250	210
T 13	800	500	280	180	150
S 1	660	400	280	150	126
2	660	400	280	150	126
3	660	400	280	150	126
4	660	400	280	150	126
A	660	400	280	150	126
5 or S5	660	400	280	150	126
6 or S6	660	400	280	150	126
B	-	400	280	150	126
C	-	-	280	150	126
D	-	-	-	150	126
E	-	-	-	-	126
TOTAL	53,720	34,960	20,520	13,640	10,376

TOTAL REQUIRED EQUIVALENT AM POWER FOR EQUAL S/N.