A DIGITAL VIDEO SYSTEM FOR THE CATV INDUSTRY

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ABSTRACT

A digital PCM system is proposed to implement long-haul video systems for the CATV industry. A comparison is made between PCM systems and wideband FM systems in terms of repeaterability versus additional occupied signal bandwidth. This is followed by a noise analysis of coaxial cable to determine the correct PCM format and bit information rate. An eighty megabit system is selected, which uses an eight bit code in a four level-eight level-eight level pulse sequence per video sample. The selected PCM system is then evaluated for its performance on both a long-haul cable system and a long-haul microwave system. Performance calculations are made on a 500 mile cable system in terms of error rate and its related video signal to noise ratio. Repeater spacings are computed for various configurations, including the replacement of the digital regenerators with analog amplifiers. Additional performance calculations are then made on a 3000 mile microwave system, including the effect of simultaneous Rayleigh fading.

Part 1: THE APPROACH

Whenever a new modulation system is proposed for a communication service, there should be some justification made for any additional level of complexity that may occur. This is especially true for a digital video system, which at first glance appears to be an expensive, complicated, "blue sky" approach to long-haul video cable networks. Indeed, the attitude of the CATV industry today toward such a system is not too unlike the telephone industry's initial reaction to the idea of a CATV industry----expensive, technically impractical, and definitely uneconomical. Historically, these objections have applied only to the time scale of implementation, rather than being an indication of the ultimate occurence of a system.

An important point to be made is that today's distribution systems have proven in the past, and will continue tobe in the future, the correct way to distribute signals within a three to five mile radius. It is improbable that ar y fundamental changes will occur at this level of distribution in the future. A digital video system would complement today's existing equipment by supplying a high quality video signal to local (five mile) distribution centers. This configuration appears quite reasonable in view of the fact that today's high quality trunk lines are at a marginal performance level beyond a distance of approximately 30 miles, especially when economic constraints are considered. This is demonstrated in Appendix 1, which shows the relationship between cost and system length for fixed performance standards. Once this basic limitation is acknowledged, alternative solutions appear in a more meaningful light.

Adapting from microwave techniques, a solution to the long-haul video problem might be to convert the video signal into a wideband FM signal. This will yield an improvement in the detected output signal to noise ratio at a cost of additional occupied signal bandwidth. However, all communication systems that improve the detected output signal to noise ratio do so by increasing the bandwidth of the transmitted information. It remains to be determined that if the bandwidth of a signal must be increased, what is the most efficient system in terms of signal to noise improvement versus increased bandwidth.

For an FM system, the improvement in S/N over an AM system may be written as:

(1) $$\frac{S/N_{fm}}{S/N_{am}} = 3k^2$$

where $k = f_d/f_m$ and

f_d = the peak deviation of the FM signal and

f_m = the highest modulation frequency of the video signal

The bandwidth of the FM signal is:

[1] Ref 10

(2) $BW = (2k + 2) f_m$ (Carson's Rule)

Now it is convenient to define $x = BW/2f_m$ which is the normalized increase in the bandwidth occupied by the signal compared to that of an AM system. Using this definition, and solving for k in equation (2), equation (1) may be rewritten as:

(3) $$\frac{S/N_{fm}}{S/N_{am}} = 3(x-1)^2$$ which is the signal to noise improvement between the two systems defined in terms of additional bandwidth occupied by the signal.

However, for a PCM system the signal to noise improvement compared to an AM system is:

(4) $$\frac{S/N_{pcm}}{S/N_{am}} = (S_c/N_c)^{BW/2f_m} = (S_c/N_c)^x$$ which is derived in Appendix 2 where S_c/N_c is the input signal to noise ratio of the PCM carrier.

This is the signal to noise improvement defined in terms of additional bandwidth occupied and the input signal to noise of the carrier. Equations (3) and (4) permit a comparison between the signal to noise improvement of an FM and a PCM system:

(5) $$\frac{S/N_{pcm}}{S/N_{fm}} = \frac{(S_c/N_c)^{x-1}}{3(x-1)^2}$$ which is the improvement of a PCM system over a wideband FM system defined in terms of additional occupied bandwidth and the input carrier to noise ratio.

The preceeding equations, when supplied with typical system parameters, quickly show the decided advantage of using a PCM system over an FM system for a given investment in occupied bandwidth. Taking $x = 3$ (which is the PCM system discussed in this paper), and an input carrier to noise ratio of 60dB we have the $S/N_{fm} = 71$ dB. If we are to permit repeatering of the signal until a detected signal to noise of 59 dB is obtained (assuming this the maximum tolerable degradation), this would leave a 12 dB margin or 2^4 maximum doubles in repeater operations which would permit a total of 16 repeaters.

However, using the same parameters for a PCM system we find that the $S/N_{pcm} = (10^6)^2 = 120$ dB which yields a margin in excess of 60 dB and would permit 2^{20} doubles or over 10^6 repeater operations. In reality, the excessive repeater capability of a PCM system can be utilized by letting the input carrier to noise ratio degrade to a lower operating level than the equivalent FM system. For example, if we assume a S_c/N_c of 40 dB for the PCM system (compared to 60 dB for the FM system), the detected signal to noise would be $S/N_{pcm} = 80$ dB which would be a margin of 2^7 or 128 repeater operations compared to the 16 possible repeater operations in an FM system of equivalent bandwidth operating with a 20 dB greater carrier to noise ratio.

A second method of utilizing the excessive repeaterability of a PCM system is to use analog repeater amplifiers instead of digital regenerators at a repeater site.

Analog amplifiers may be used in cascade (reamplifying the digital signal) until the S_c/N_c has degraded to 40 dB (using the previous example at the higher operating level of 60 dB S_c/N_c and cascading analog amplifiers until 20 dB of degradation has occured.) At this point a digital repeater would be used to regenerate the signal, after which it may be decoded for distribution, or followed by additional analog repeaters.

Although the preceeding advatages of a PCM system over a wideband FM system are true for all types of PCM systems, several parameters must be evaluated to determine which PCM system would permit optimum performance for video when used with cable transmission. These include the "quantizing noise" which is the intrinsic noise level that occurs when a signal is divided into discrete steps, and the consideration as to whether the system is to be used at baseband on the cable or multiplexed at higher frequencies on the cable similar to the present AM systems. Appendix 3 contains a derivation of the quantizing noise that occurs in signal quantizing, and tabulates the results in terms of the number of levels that the continuous signal is quantized. Table 1 in Appendix 3 shows that if 256 levels are used, the intrinsic quantizing noise is reduced to 59 dB below signal level. The quantizing noise of a PCM system occurs at the original coding of the signal and is independent of the number of repeater operations. In fact, nearly all normal video specifications are independent of the number of repeater operations and dependant solely on single hop performance. These include differential gain, differential phase, frequency response, square wave tilt, and video bounce. Once the video signal has been placed in a PCM format, the only signal degradation that may occur are the errors that occur in the decision process of determining the correct occurence of a pulse. These "errors" are then reflected in the detected output signal to noise ratio of the video signal. The error rate of the PCM signal is proportional to the product of the bit rate and the probability of error. Appendix 4 gives a derivation of the probability of error in terms of input carrier to noise. Figure 1 in Appendix 4 graphs the probability of error versus carrier to noise ratio for binary, quaternary, and octenary level pulses. These curves will be used later to determine the error rate of the PCM system which may then be related to final output signal to noise level.

Another fundamental consideration of the PCM system is the determination of which portion of the cable spectrum it will occupy, especially under the conditon of transmitting multiple channels. Appendix 5 contains a cost comparison between single and multicore cables. It demonstrates that for a multi-channel system, it is no more expensive to use one small cable for each channel than it is to use one large cable with frequency division multiplexing. Transmission loss requirements dictate that multichannel PCM systems be transmitted on a multicore cable with each channel occupying the baseband (lower end) portion of the cable spectrum

THE SYSTEM

Fundamental to the formulation of a PCM system is the determination of the data rate required for the system. The data rate is composed of two factors: thesampling rate f_s multiplied by the number of bits per sample (= $\log_2 N$ where N is the number of quantizing levels). The minimum sampling rate required is:

$$f_s = 2f_m \qquad \text{(Nyquist sampling rate)}$$

where f_m is the highest frequency component of the baseband information. For color video only, f_m of 4.2 mHz would be required. However, if the system is required to caray inter-carrier sound, an f_m of 4.525 mHz would be appropriate. If a pilot is to be placed in the baseband above the video information, it would not be unreasonable to consider f_m extending to 5 mHz. This would imply a sampling rate of 10 mHz. Since the bandwidth required is directly proportional to the sampling rate, it is advisable to keep the sampling rate as close to the Nyquist rate as possible.

The second factor in the data rate is the number of bits per sample, which in turn is related to the number of quantizing levels used. The determination of the required number of quantizing levels is reached by a consideration of the delivered picture "quality". Appendix 3 gives the relationships between the number of quantizing levels and the broadband signal to noise ratio due to "quantizing" noise.

On the basis of signal to noise only, Table One in Appendix 3 would indicate that a 6 bit code (64 levels) would yield a 47 dB S/N ratio, which would be adequate for many purposes. However, the implementation of a 6 bit code, when evaluated by subjective tests, indicate a "contouring" phenomenon due to insufficient quantizing levels. Contouring transforms continuous shading into a number of discrete steps. Contouring effects are a coherent type of interference, and therefore require a higher signal to noise ratio than would ordinarily be required for random noise. This is analogous to the cross modulation requirements of a video signal being much higher than the tolerable broadband signal to noise ratio.

If random noise is introduced to reduce the contouring effects, a usable picture may be transmitted with a 6 bit code, but the system will not meet commercial broadcast standards. By using an 8 bit code, contouring effects become negligible, and the system may now be used for other purposes, such as multichannel telephone. From Table One in Appendix 3, this corresponds to a broadband signal to noise ratio of 59 dB.

With the selection of an 8 bit code (256 levels) and a 10 mHz sampling rate, the system data rate is 80 megabits/sec. For a binary system, this would imply a transmitted bandwidth of 40 mHz, with eight pulses per sample. If a quaternary (4 level) system were used, the required bandwidth would be reduced to 20 mHz, and the number of pulses per sample would be four. Appendix 6 shows a noise analysis for a digital cable transmission system. The results in Appendix 6 indicate that a satisfactory pulse format would be three pulses per sample in a 2-3-3 bits per pulse arrangement. (This means a four level pulse followed be two eight level pulses for each sample.) This format also has the advantage of reducing the required bandwidth to 15 mHz, as well as having certain advantages in the area of synchronization, due to unique transition possibilities.

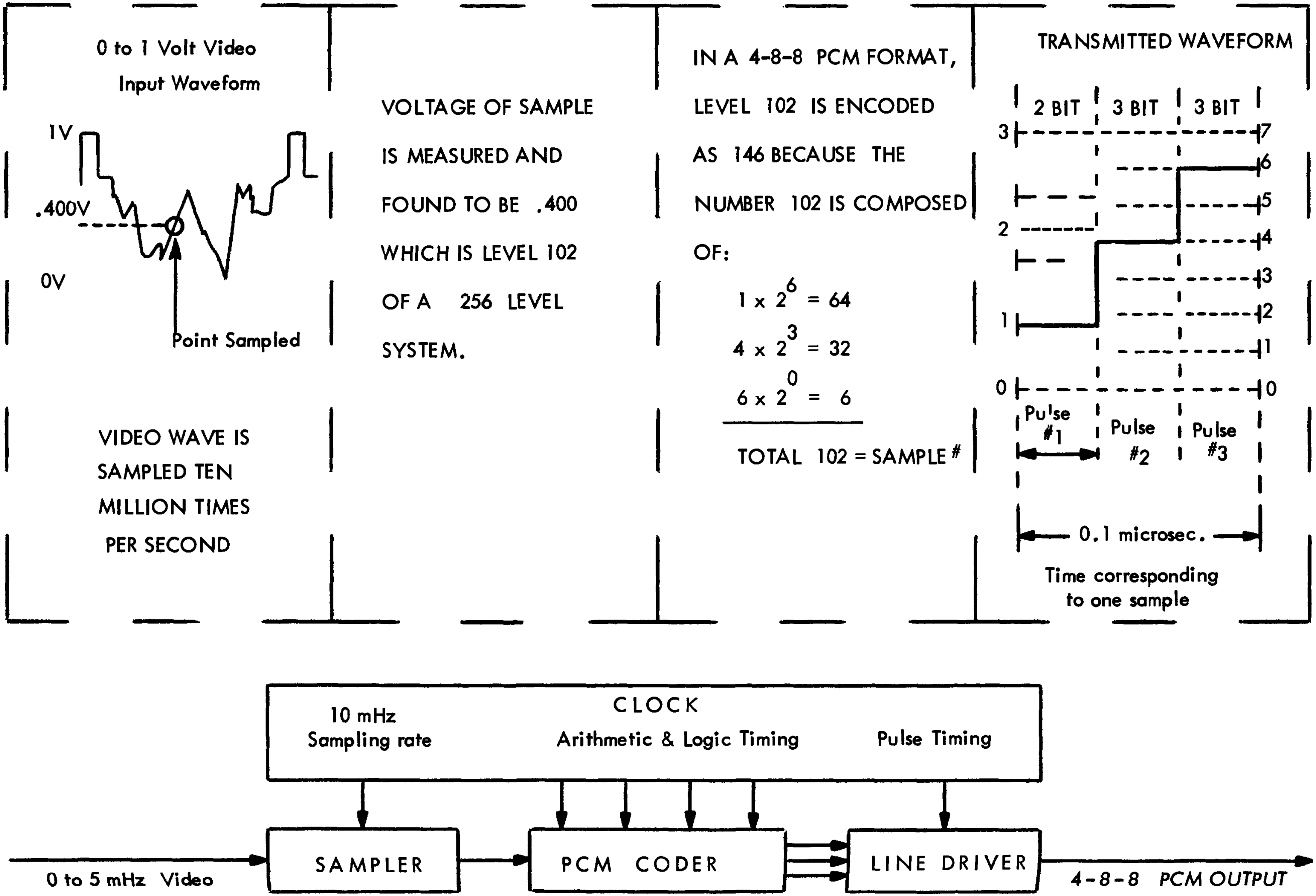
The synchronization problem in a PCM system is the determination of the correct weighting of the received pulses, that is, recognizing the sequence in which they were generated. This may be achieved by transmitting a unique code group periodically, or for a wired system, a CW clock may be passively added at the transmit end and recovered ahead of the first active device at the receive terminal. Where nonuniform coding is employed (such as the 2-3-3 bit format) synchronization may be achieved by recognizing the unequal transitions between the four and eight level pulses.

For a cable transmission system, it is desireable to employ a pulse format which does not necessitate the transmission of low frequency signals. Coaxial cables exhibit very uniform phase characteristics above frequencies of 100 kHz. Also, transformer coupling elements are more easily achieved if the number of decades of frequency range covered is held to three or four. Another advantage of not using the very low end of the cable spectrum is that hum pickup can be ignored and power for the repeater stations can be placed directly on the cable.

An eight bit code with a 2-3-3 bit format would require a pilot signal level approximately 30 dB below the video signal to ensure that the third pulse of the sample group will periodically oscillate through all possible values, even in the absence of an input video signal. By using a pilot at 4.7 mHz, it is possible to restore frequency components below 300 kHz. The pilot also permits AGC action in the receiver by guaranteeing frequent occurrence of maximum and minimum level pulses.

The preceding discussion of the PCM system is best summarized by the drawing on page 12. This drawing shows the input video signal being sampled at a rate of 10 million times per second. These output samples (which are directly proportional to the input video signal) are then quantized into 256 possible levels. Whichever of the 256 levels is selected for a sample becomes processed into a three pulse format where the first pulse can have four possible levels and the second two pulses can have eight possible levels (2-3-3 bit PCM format). The processing of a single sample is illustrated on page 12 in the drawing above the block diagram.

FUNCTIONAL DIAGRAM OF 4-8-8 PCM SYSTEM



Part 3: PERFORMANCE CALCULATIONS

Consider a single video channel to be transmitted on a 500 mile cable system, using the previously described PCM system. Assume a peak power of .1 Watt is the output at the transmitter or at any repeater site. The noise figure of the receiver and any repeater amplifier (either analog or digital) is taken to be 10 dB. From Appendix 6, the highest frequency to be transmitted down the cable is 15 mHz (F). For a video signal to noise at the end of the system of 56 dB (that is, line contributed noise equal to quantizing noise), the video signal to noise at the receiver due to error rate performance of the system must be 59 dB. The error rate for a 59 dB video signal to noise may be computed using Equation (8) of Appendix 4 or interpolated from Table Two in Appendix 4. In either method, the corresponding error rate for a 59 dB signal to noise of the video signal is 1.3×10^{-3}. These errors can be assumed to be equally contributed by each repeater of the system.

Conservatively, assume there will be 500 repeater sites in the system. (This is a pessimistic number, as the results will show that 114 repeaters are required.) This would mean that each repeater would be permitted to contribute an error rate of 2.6×10^{-6}. All of these errors will occur in the two eight level pulses (since the four level pulse is essentially operating at a higher signal to noise ratio).

The required carrier to noise ratio at the input of a regenerator to produce 2.6×10^{-6} error rate may be found by reading Figure One of Appendix 4 or interpolating Table Two of Appendix 4. The required carrier to noise ratio is 38 dB, and for a .1 Watt signal level, this defines the noise power out of a repeater to be -18 dBm. By using Equation (5) of Appendix 6, the permissible cable loss between repeater sites is found to be 61 dB at a frequency of 10 mHz. The actual distance will depend on the diameter of the cable used. For representative cable of different diameters, the results may be tabulated as follows:

Cable Diameter	Atten. dB/mile @ 10 mHz	Repeater Spacing	# Repeaters
.4 inches	13.7	4.4	114
.3 inches	18.3	3.3	150
.2 inches	27.4	2.2	225
.1 inches	54.8	1.1	450

For a cable diameter of .4 inches (.26 dB/100 ft @ 10 mHz) the repeater spacing will be 4.4 miles, which would require 114 repeaters. This shows the noise assumptions to be quite conservative, since even .1 inch cable will require less than 500 repeaters.

The preceding calculations are predicated on using digital regenerators at all repeater sites. It is interesting to investigate the possibility of adding analog amplifiers between the digital regenerators of this system. If n analog amplifiers are placed between each pair of digital regenerators, the attenuation spacing, K_d, for the digital system must be reduced by $\frac{3\log_2(n+1)}{n}$. The total system attenuation for N digital repeaters was:

$A_{system} = N K_d$. With analog amplifiers inserted in the system, this becomes:

$$A_{system} = N \cdot \left[\left(K_d - 3/n \cdot \log_2(n+1) \right) \cdot (n+1) \right] \quad (1)$$

For the smallest cable on the tabulation of page 14, the system loss for 500 miles was:

$A_{system} = 450 \times 61 = 2740$ dB . Inserting seven analog amplifiers between each regenerator we have: (Using Equation (1) above)

$A_{system} = 21.5 \times 10^4$ dB and the system length is 3900 miles. This indicates that the limit to this technique will be set by the difficulties of phase equalization and gain control, rather than any noise considerations.

In a similar manner, we could have taken the original 500 mile system and replaced all of the digital regenerators with analog amplifiers. For .4 inch cable, the system would have required 180 analogue amplifiers at a spacing of 2.7 miles. Again, this illustrates that thermal noise will not be a limiting constraint on the PCM system.

To allow a comparison between cable and radio systems, apply the 2-3-3 bit PCM format to a microwave transmitter. Let a combined amplitude and phase shift keying modulation system be used. (Four phase-two amplitude with no zero level). One phase position of the four level pulse may have itself identified by being transmitted at minimum rather than maximum level. This will allow phase identification for coherent detection. The signal may be filtered to produce a vestigial sideband spectrum which may be transmitted in a 20 mHz radio channel. The probability of error for such a channel (P_e) when subjected to Rayleigh fading is given approximately by:[2]

$$P_e = \frac{5}{C/N}$$ for C/N greater than 10 ~~(After Stein, Ref. 9)~~

(13 dB worse than a binary DSB-PSK system)

For a 500 mile radio system consisting of 20 repeaters at a 25 mile spacing, and a system error probability requirement of 1.3×10^{-3} (as for the cable system), the individual repeater links must contribute only $P_e = 1.3 \times 10^{-3} / 20 = .65 \times 10^{-4}$.

The required carrier to noise ratio into the regenerator site is:

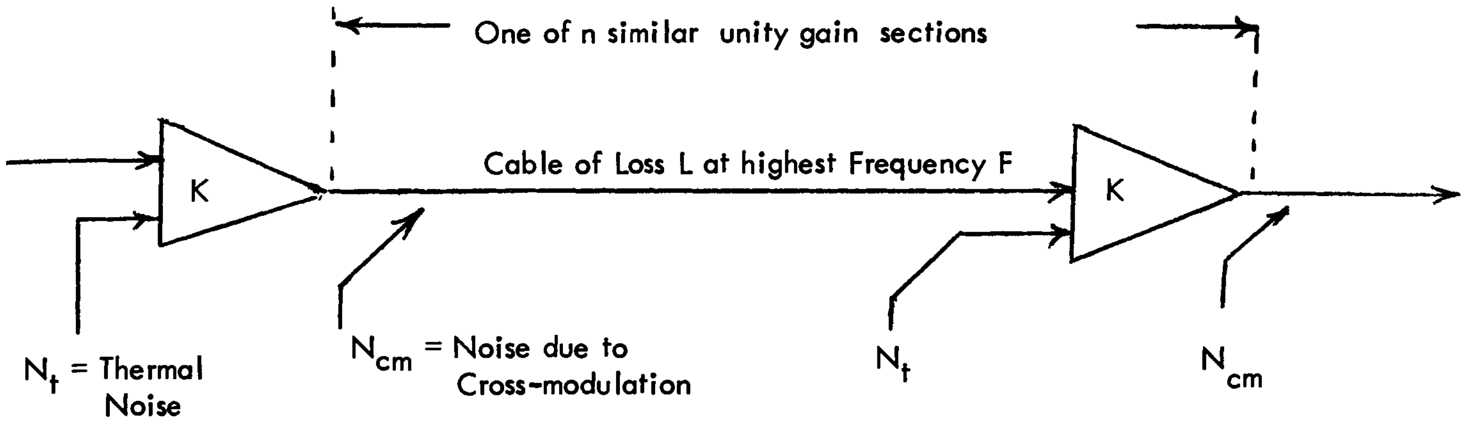
$C/N = 5/P_e = 7.7 \times 10^4$ or 48.6 dB .

Placing the channel at 12.5 gHz with antennas of 4 ft. diameter yields a space loss of 146.5 dB and an antenna gain (2 antennas) of 83 dB. With a 12 dB receiver noise figure the tangential sensitivity is -89 dBm and the required input carrier to noise is 48.6 dB above this or -40.4 dBm. Adding this level to the path loss minus the antenna gain yields a transmitter power requirement of +22.6 dBm which is well within the state of the art for solid state devices. The corresponding calculations for 120 repeaters (3000 mile system) yield a transmitter power requirement of +30.6 dBm which is approximately state of the art for solid state sources at 12 gHz at the present time. By increasing the antenna size, a satisfactory 3000 mile system which operated in the presence of simultaneous Rayleigh fading can be achieved.

[2] After Stein, Ref 9

APPENDIX ONE

LIMITING CONSTRAINTS ON EXISTING CABLE SYSTEMS



The preceeding drawing shows one of n unity gain sections of a cable transmission system. Let the video signals be applied in a frequency division multiplex arrangement using a modulation system comparable to conventional broadcast standards. Let the cable loss for the highest frequency channel be L and the gain of the amplifier (including a built in cable equalizer) be K where

(1) $\quad K = 1/L$

Thermal noise of power density N_t ($N_t = 4 \times 10^{-15} f$ watts per megacycle where f is the noise factor of the amplifier input) is added at the input of each amplifier and cross modulation noise power N_{cm} (due to amplifier non-linearities) is added at the output of each amplifier. $N_{c\,n}$ may be assigned a spectral density of watts/ mHz where:

(2) $\quad N_{cm} = MW_o^3$ where W_o is the output power per channel and M is an amplifer constant derived by subjective test. Essentially, M corrects for the fact that the well correlated third order cross modulation products cause much more picture degradation than would a similar amount of white noise. M may be written numerically by:

(3) $\quad M = 1/BCW_t^2$ where B = the channel bandwidth in megacycles
C = S/N (power ratio) for just visible thermal noise and
W_t = Single channel output power for just visible cross modulation interference on a single amplifier test.

For a cascade of n amplifiers the equivalent noise power out of the last amplifier is:

(4) $W_n = nB(N_{cm} + KfN_t) = nB(MW_o^3 + KfN_t)$ where W_n is the total noise power out of the system. The signal to noise ratio W_o/W_n is:

(5) $S/N = W_o/nB(fKN_t + MW_o^2)$

An optimum system will operate at a level where thermal noise and cross modulation effects cause equal degradation. At this level of operation:

(6) $fKN_t = MW_o^3$ or $W_o = (fKN_t/M)^{1/3}$

Substituting this value of W_o in (5) we have

(7) $S/N = 1/2nB(Mf^2K^2N_t^2)^{1/3}$

When an amplifier noise factor and output capability M is known, the maximum number of amplifiers in cascade for a given output signal to noise ratio may be determined as

(8) $n = AK^{-2/3}$ where $A = \dfrac{1}{2B(S/N)\sqrt[3]{M(fN_T)^2}}$

The length of the whole system in terms of total system gain G is :

(9) $G = K^n = K^{AK^{-2/3}}$

To find the maximum value of G (and therfore the longest system for a given cable loss) we differentiate (9) and set $dG/dK = 0$. Hence,

(10) $dG/dK = 1 - 2/3 \log_e K = 0$ or $K = e^{3/2}$ or 6.5 dB for amplifier gain.

Substituting in (8) yields the number of amplifiers for the longest system as

(11) $n_{max} = AK^{-2/3} = A/e$ and the total system gain is:

(12) $G_{max} = K^n = (e^{3/2})^{A/e} = 2.33A$ dB Where A is defined in Equation (8)

If representative numbers are placed in Equation (12) (M has a range of 20 to 200) we find the loss of the maximum cable system to be in the range of 800 to 1200 dB, depending on the particular amplifier chosen. This implies that the only method of extending the cable system is to arbitrarily increase the size of the cable (provided the best available amplifier has already been used). This is the least attractive method of increasing system length since the cost of the cable per unit length is proportional to the square of the diameter. It is convenient to define a cable figure of merit s:

(13) $s = Ld/\text{length}$ where L is the loss at Channel 13 and d is the diameter of the cable (inche and the length is measure in miles. For some of the new foam dielectric cables, s has a value of 27.5 dB/mile/inch diameter of cable. This would predict a maximum system length in the range of 30 to 40 miles if one inch diameter cable were used. However, in this development there was no provision for operating level tolerance, echo distortion noise due to vswr, and cumulative frequency response effects, all of which tend to shorten the maximum operating length.

At the point where the only alternative to increasing system length is to increase the diameter of the cable, the total cable cost of the system is:

(14) $Q = pd^2 \times (\text{system length})$ Where Q is the total cable cost and p is a constant equal to the cost of one mile of one inch diameter cable.

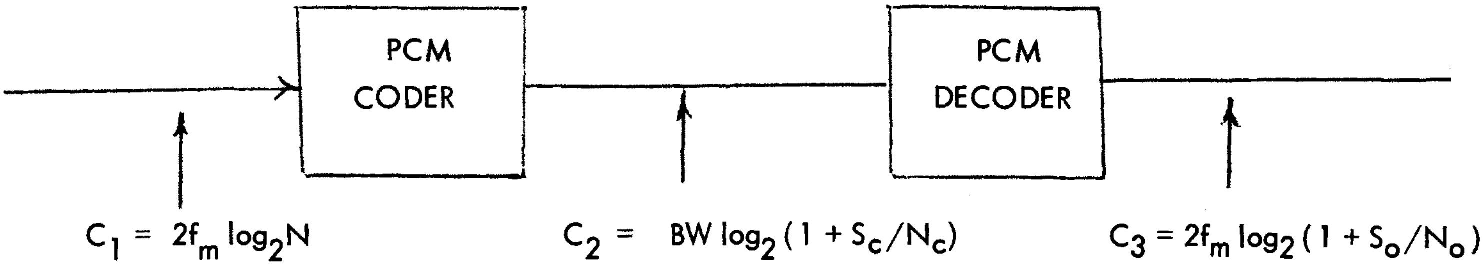
Recognizing that $d = s/L \times (\text{system length})$ and that we are operating in a range where $L = 2.33A$ (maximum system length) we have:

(15) $Q = p(s/2.33A)^2 \times (\text{system length})^3$

The results of Equation (15) may be interpreted as follows: For a short cable system (where the diameter of the cable need not be increased), the cable cost is proportional to the first power of system length. Beyond 200 to 300 dB (where cable size can be compromised by using additional amplifiers) the system designer may accept an increase in the exponent of system length (to a squared function) to make possible the use of fewer amplifiers and to provide gain margin for maintanence ease. However, in the range of 800 to 1200 dB (where the only alternative is to increase cable diameter), the cable cost starts to vary as the third power of the system length, because the limit on the quality of amplifiers has been reached and the only way on increasing system length (or level margin) is to increase the diameter of the cable used.

APPENDIX 2

DERIVATION OF SIGNAL TO NOISE IMPROVEMENT IN A PCM SYSTEM



The above diagram shows the information capacity C of a PCM system that accepts an N level quantized signal and coverts it into a PCM format for transmission, and reconverts the signal back to its original input after transmission. C_1 is the information rate expressed in terms of the highest baseband frequency f_m and the number of quantization levels N.[3] C_2 and C_3 are the information capacity of the system expressed in terms of bandwidth BW and signal to noise ratio (Shannon's Law)[4] S_c/N_c is the carrier to noise input to the decoder, and S_o/N_o is the signal to noise ratio after the decoding process.

If we postulate that there is to be no information lost in the system (i.e it will be operated at a high enough signal to noise ratio to maintain the required information capacity), the information capacity at all points in the system must be equal, and C_2 must be equal to C_3. Therefore:

(1) $$C_2 = BW \log_2 (1 + S_c/N_c) = 2 f_m \log_2 (1 + S_o/N_o)$$

which may be written as:

(2) $$(1 + S_o/N_o) = (1 + S_c/N_c)^{BW/2f_m}$$

For signal to noise ratios much greater than one, (2) may be expressed as:

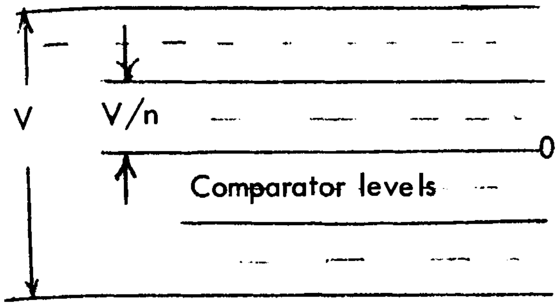
(3) $$S_o/N_o = (S_c/N_c)^{BW/2f_m}$$

3 Ref 6 Bibliography

4 Ref 11 Bibliography

APPENDIX 3

DERIVATION OF QUANTIZING NOISE IN A PCM SYSTEM



Consider a peak to peak signal V volts to be quantized into n levels. The spacing between levels is V/n volts. The quantizing comparators would be set at $\pm V/2n$, $\pm 3V/2n, \ldots (2n-1)V/2n$. Each quantized output level represents all signal values in the range of $+V/2n$ to $-V/2n$ about its value. The difference between the quantizing level and the true signal value is the error introduced into the system. Assuming that over a long perido of time all values of signal in the uncertainty range of $+V/2n$ to $-V/2n$ are equally likely, the signal may be described as $A_i + E$ where A_i is the level being transmitted, and E represents the error voltage between the actual signal and its quantized equivalent. From before, E must fall between the range of $+V/2n$ to $-V/2n$. We may then write the mean squared value of E as:

$$\overline{E}^2 = n/V \int_{-V/2n}^{V/2n} E^2 \, dE = V^2/12n^2$$

The rms value of the error is then $V/n\sqrt{12}$, and since the peak to peak signal is V, the peak to peak signal to the rms error noise (quantizing noise) is $n\sqrt{12}$ in voltage ratio or $12n^2$ for the corresponding power ratio. This is the signal to "quantizing noise" ratio, and it may be made increasingly large by arbitrarily increasing the number of levels.

TABLE ONE

QUANTIZATION S/N vs NUMBER OF LEVELS

Quantization S/N dB	# of levels, n
17	2
23	4
29	8
35	16
41	32
47	64
53	128
59	256

APPENDIX 4

ERROR RATES VERSUS VIDEO SIGNAL TO NOISE

Consider a 1V p - p video signal transmitted in a four level-eight level-eight level pulse sequence per sample at a rate of 10^7 samples per second. Let the output level of the PCM transmitter be peak power limited at a level of $P = V_o^2$. The the power for any one step in an n level pulse is: [6]

(1) $P_n \ (V_o/n-1)^2$ and the probability of error (P_e) is:

(2) $$P_e = \frac{2(n-1)N}{\sqrt{2\pi}} \cdot \exp\left(-V_o^2 / 8(n-1)^2 N^2\right)$$

where N is the rms noise voltage, and V_o/N is the signal to noise ratio in terms of p-p signal to rms noise.

Equation (2) may be used to obtain separate expressions for the probability of error for a binary, quaternary, and octenary level pulse:

(3) Binary: $P_{e_2} = 0.8(N/V_o) \cdot \exp(-0.125 V_o^2 / N^2)$

(4) Quaternary: $P_{e_4} = 7.2(N/V_o) \cdot \exp(-0.0139 V_o^2 / N^2)$

(5) Octenary: $P_{e_8} = 392(N/V_o) \cdot \exp(-0.00255 V_o^2 / N^2)$

Equations (3), (4), and (5) are presented graphically in Figure One which shows the relation ship between error rate, input signal to noise ratio.

It is useful to be able to convert readily between error rate, carrier to noise ratio, and output video signal to noise ratio. These relationships are dependent on the type of PCM format that is used. For the four level-eight level-eightlevel pulse sequence described in the text the following relationships apply:

If any of the three pulses of the code (4-8-8) are misread due to the presence of noise, there will be a noise voltage included in the decoded output. The rms value of this noise voltage will depend upon the probability that an error will occur and the noise power introduced

6 Ref 3 Bibliography

by a single error in that pulse. Since the pulses are weighted, the noise contribution of their respective errors is unequal. If normalized values of V_o = 1V p-p and Z_o = 1 ohm, we may add the noise powers contributed by each of the pulses to produce:

$$(6) \qquad W_n = (1/4)^2 \cdot P_{e4} + (1/32)^2 \cdot P_{e_8} + (1/256)^2 \cdot P_{e8}$$

where W_n is the noise power and P_{e4}, P_{e8} are the probability of error in a four level and eight level pulse respectively. To correct for second order effects, we should add the noise power contributed by 2 level errors on the eight level pulses. A 2 level error on an eight level pulse is equivalent (in probability) to a single level error on the four level pulse and has twice the weighted error of a single error that occurs on the same pulse. From this we have:

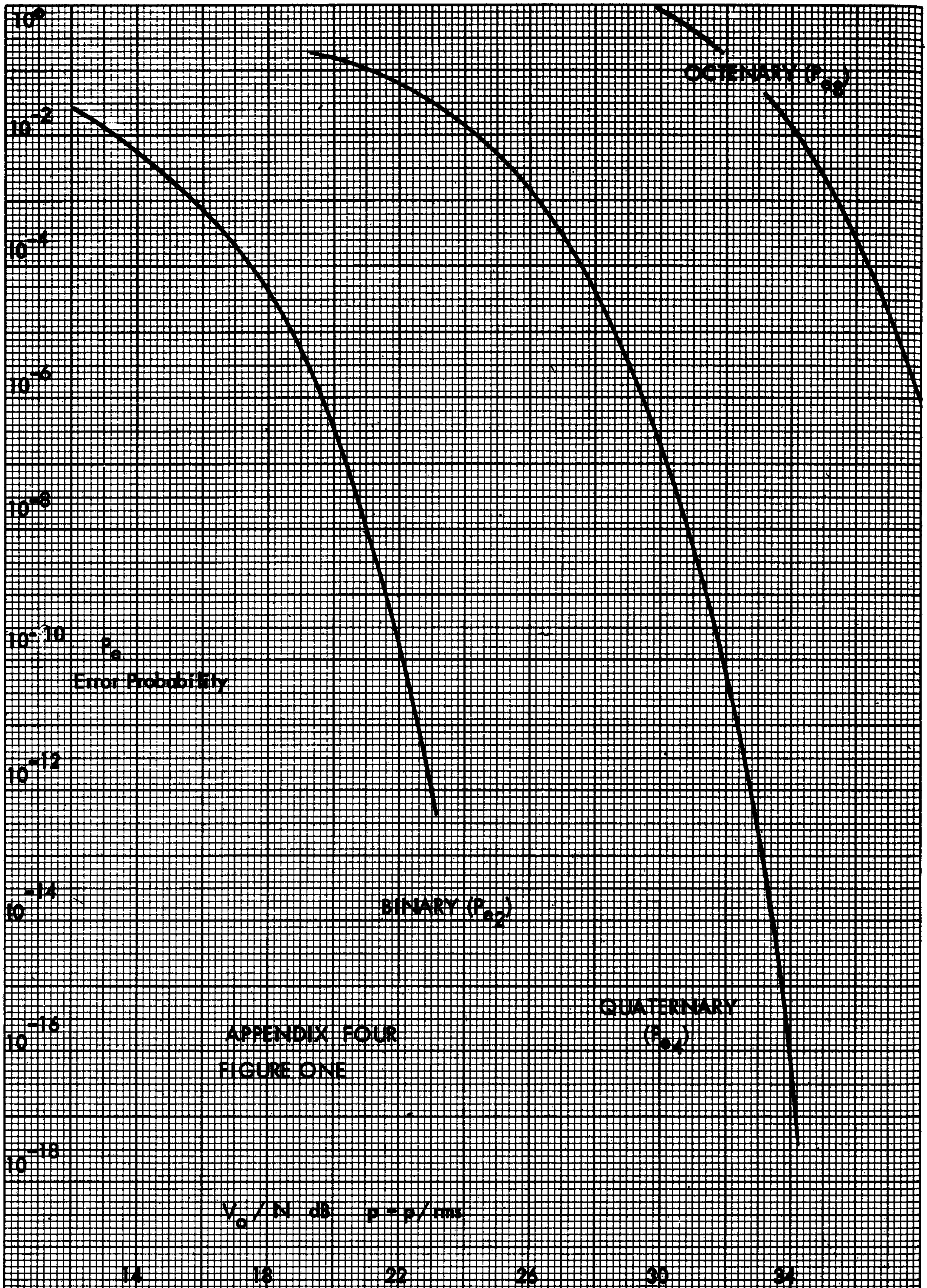
$$(7) \qquad W_n = (1/4)^2 \cdot P_{e4} + (1/32)^2 \cdot P_{e8} + (1/16)^2 \cdot P_{e8} + (1/16)^2 \cdot P_{e4} + (1/128)^2 \cdot P_{e4}$$

which upon collecting terms becomes:

$$(8) \qquad W_n = 6.64 \times 10^{-2} \cdot P_{e4} + 9.78 \times 10^{-4} \cdot P_{e8}$$

Equation (8) is useful in converting noise power into error rates directly, provided W_n is expressed directly in watts. By using equation (8) and Figure One, we are able to compute the video signal to noise in terms of input carrier to noise. This has been done, and the results are tabulated in Table Two which also includes the corresponding error rates for the four and eight level pulses.

The results of Table Two are shown graphically in Figure Two which shows the output video signal to noise ratio in terms of input carrier to noise ratio for the 4-8-8 code format. Notice that for high signal to noise ratios (carrier to noise) that the video signal to noise is limited by the intrinsic quantizing noise of the PCM system. As the carrier to noise degrades, so does the video signal to noise, first due to errors in the eight level pulses, and finally due to errors in the four level pulses.



10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
P_e
Error Probability
OCTENARY (P_{e8})
BINARY (P_{e2})
QUATERNARY (P_{e4})
APPENDIX FOUR
FIGURE ONE
V_o / N dB p - p/rms
14
18
22
26
30
34

TABLE TWO

CARRIER TO NOISE VERSUS VIDEO SIGNAL TO NOISE

C/N dB	P_{e4}	P_{e8}	Video S/N dB
23	3.25×10^{-2}	1	24.8
24	1.38×10^{-2}	1	27.2
25	5.02×10^{-3}	1	28.9
26	1.43×10^{-3}	1	29.5
27	3.01×10^{-4}	1	30.0
28	4.32×10^{-5}	1	30.0
29	3.81×10^{-6}	1	30.0
30	2.08×10^{-7}	9.68×10^{-1}	30.5
31	5.60×10^{-9}	4.08×10^{-1}	34.0
32	5.03×10^{-11}	1.75×10^{-1}	37.7
33	1.84×10^{-13}	5.62×10^{-2}	42.5
34	9.90×10^{-17}	1.29×10^{-2}	49.2
35	1.10×10^{-20}	2.23×10^{-3}	56.5
36		2.47×10^{-4}	66.0
37		1.52×10^{-5}	78.0
38		5.04×10^{-7}	93.0
39		7.47×10^{-9}	111.
40		3.29×10^{-11}	135.

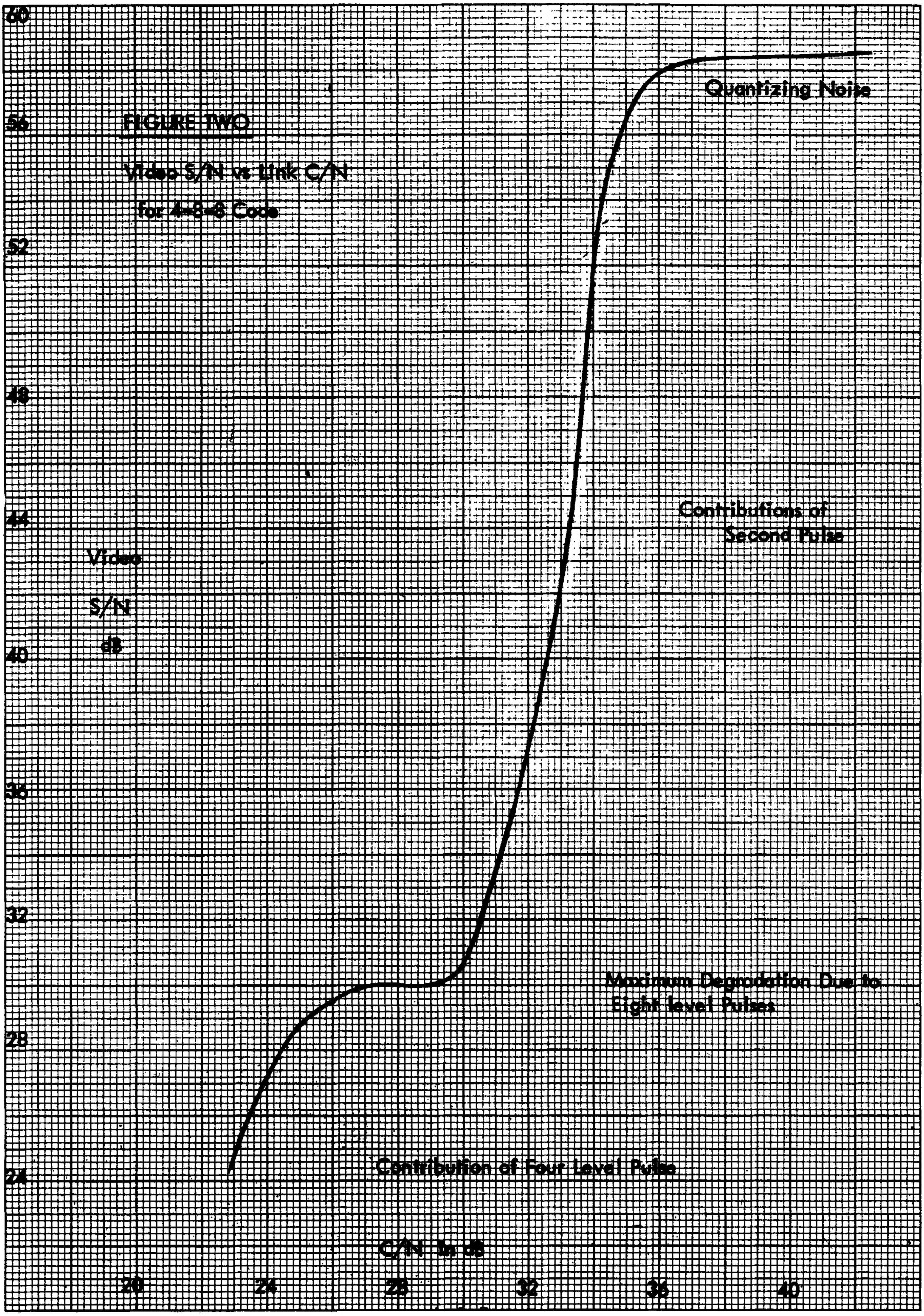


FIGURE TWO
Video S/N vs Link C/N
for 4-8-8 Code
Quantizing Noise
Contributions of
Second Pulse
Maximum Degradation Due to
Eight level Pulses
Contribution of Four Level Pulse
Video
S/N
dB
C/N in dB
60
56
52
48
44
40
36
32
28
24
20
24
28
32
36
40

APPENDIX 5

COST COMPARISON OF SINGLE VS MULTICORE CABLES

The cost of a coaxial cable may be represented by the expression

(1) $C = k_c D^m$ where C = cost per unit length of cable
k_c = constant determined by type of cable construction
D = the diameter of the cable
m = a number approximately equal to 2 depending on the processing costs of the cable. If the cost of the cable were all material and no processing costs were involved, m would be 2 since the material content per unit length varies with the square of the cable diameter.

The attenuation of a cable where dielectric losses can be neglected is :

(2) $A = k_a R$ where A = attenuation in nepers per unit length
k_a = a constant determined by the characteristic impedance
R = conductor resistance per unit length

The conductor resistance per unit length is:

(3) $R = \dfrac{k_r\sqrt{f}}{D}$ where k_r is a constant determined by the conductor material and f is the frequency in Hz.

Substituting (3) into (2) and solving for D, the cable diameter, equation (1) may be written as:

(4) $C = k_c(k_a k_r\sqrt{f}\ A^{-1})^m$

Now if N channels of bandwidth B are to be transmitted on Q cables, the highest frequency that must be transmitted is:

(5) $f = NB/Q$

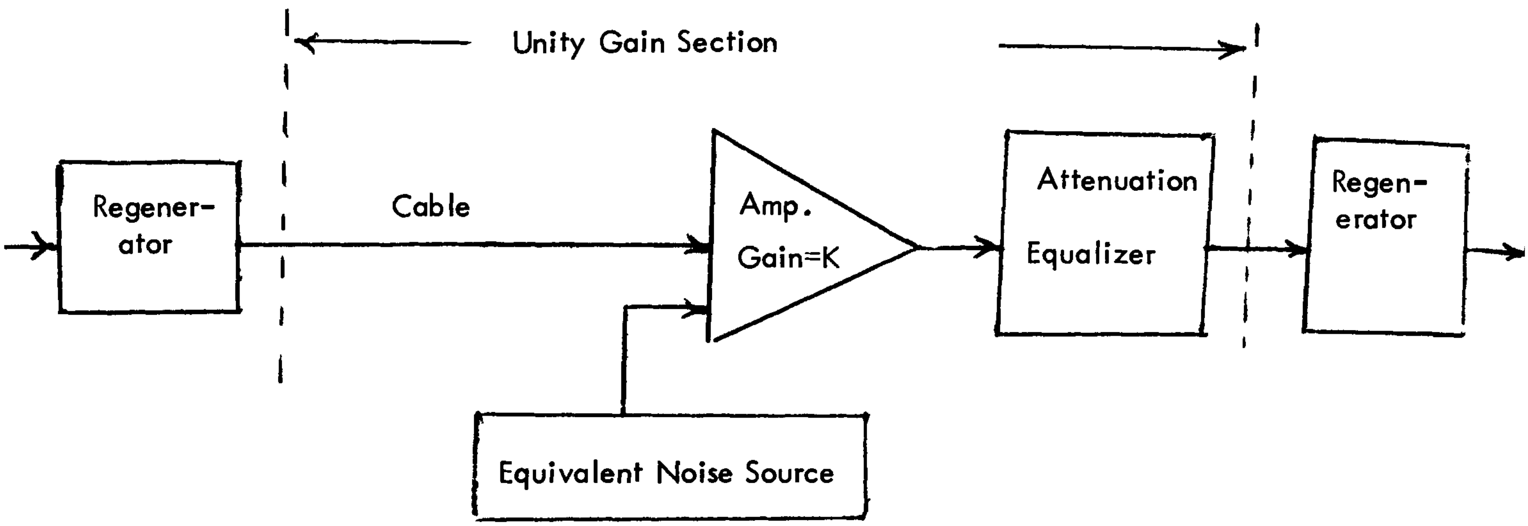
Substituting (5) into (4) and recognizing the total cost C_t is Q times C , we have for m approximately equal to 2 :

(6) $C_t = k_c NB(k_a k_r/A)^2$ which is independent of the number of cables used.

The interpretation of the above result is that for a multi-channel system, it is no more expensive to use one small cable for each channel than it is to use one large cable (whose cross-sectional area is equal to the sum of all the small cables) with frequency division multiplexing. For shorter cable runs, the cost of the multiplexing equipment would be the controlling element, whereas for long runs, the cost of single channel re-amplifying equipment becomes a consideration. In all cases, the multi-conductor cable has the reliability advantage in that a single channel failure does not interupt service on the other channels.

APPENDIX SIX

NOISE IN A DIGITAL CABLE REPEATER



The preceeding drawing shows a typical digital repeater section, with the regenerator driving the coaxial cable whose loss is given by:

(1) $A = \exp(-s\sqrt{f/f_o})$ where A is the cable attenuation and f is the frequency, and s is defined to be:

$$s = (1/4.343)(\text{loss in dB of the cable at frequency } f_a)$$

The output of the cable is combined with a white noise signal at the input of a flat amplifier. The noise source provides an amplifier input of power density $N_d = (4 \times 10^{-15})(d)$ watts per megacycle, where d is the noise factor of the amplifier. The gain of the amplifier is a factor K.

Following the amplifier, there is an equalizer for cable attenuation. Its Loss L may be described by the law:

(2) $L = \exp(s\sqrt{f/f_o}) \times \exp(-s\sqrt{f/f_o})$

For a unity gain system, we can set $K = \exp(s\sqrt{F/f_o})$ where F is the maximum frequency to which the system is to be equalized.

For a unity gain repeater section A x K x L = 1 over the frequency range from 0 to F. The noise out of the unity gain section in an incremental frequency range, df, is:

(3) $dW_n = N_d K L df = N_d \exp(s\sqrt{f/f_o}) \cdot df$ where W_n is the noise power in the section.

Therefore, W_n may be written as:

(4) $$W_n = \int_0^F \exp(s\sqrt{f/f_o}) \cdot df = (2N_d f_o/s^2)\left[\exp(s\sqrt{F/f_o}) \cdot (s\sqrt{F/f_o}) + 1\right]$$

Where the cable attenuation is high, $s\sqrt{F/f_o}$ is much greater than 1, and Equation (4) becomes:

(5) $$W_n = (2N_d\sqrt{f_o F}/s) \cdot \exp(s\sqrt{F/f_o}) = 2N_d F \cdot \frac{\exp(x)}{x}$$ where $x = s\sqrt{F/f_o}$

and $\exp(s\sqrt{F/f_o}$ is the gain of the repeater amplifier. Equation (5) gives the noise power in a digital cable repeater section. The problem remains to determine an upper bound for F interms of the information rate and the number of levels used in the PCM system.

Assume that the signal to be transmitted is a pulse of n possible levels, and the system is peak power limited to W_s watts. The power change represented by one level is then:

(6) $W_l = W_s/n-1$ and the signal to noise out of the repeater link (S/N) is:

(7) $$S/N = W_l/W_n = \frac{W_s/n-1}{2N_d F \cdot \frac{\exp(x)}{x}}$$ where S/N is the signal power for one level change divided by the total channel noise power.

For a give information rate R and a pulse repition rate of 1/T we have:

(8) $R = (1/T)\log_2 n$ and by substituting F = 1/2T where F is the maximum frequency,

(9) $F = R/2\log_2 n$ which may be substituted into Equation (7) to yield:

(10) $$S/N = \frac{W_s \log_2 n}{4RN_d(n-1)} \cdot \frac{y}{\exp(y)}$$ where $y = s \cdot \sqrt{\frac{R}{2f_o \log_2 n}}$

Equation (10) permits the calculation of the repeater spacing in terms of cable loss s versus the number of levels used for fixed system parameters. For the PCM system discussed in the text, we have as parameters:

W_s = .1 Watts
R = 8×10^7 bits per second
N_d = 10^{-19} watts per cycle (14 dB noise figure)
S/N = 200 (10^{-12} error rate)
f_o = 10 mHz

By selecting f_o = 10 mHz, we will obtain our results directly in terms of 10 mHz cable loss. The substitution of the preceeding values into Equation 10 is tabulated as follows:

n	s	s in 10 mHz cable loss (dB)
2	9.77	42.4
4	13.5	58.6
8	16.1	69.9
16	18.1	78.4
32	19.7	85.3
64	20.8	90.3
128	21.7	94.2

The above tabulation, which is for a fixed error rate (10^{-12}) shows that if thermal noise were the only consideration, there would be no penalty in raising the number of levels transmitted per pulse. This means that thermal noise effects are not a consideration in determining the number of levels used in a PCM format, and that other effects such ringing, overshoot, and delay distortion should be considered.

The 4-8-8 level code selected for the 80 megabit system represents a compromise between n =4 (4-4-4-4)of the above tabulation (which would occupy a bandwidth of 20 mHz and n = 8 (which would occupy a bandwidth of 13.3 mHz.)

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DISCUSSION

Mr. Kirk: Are there any questions?

Mr. Lowe: Mr. Lowe, Stanford Research Institute. Did you use any linear quantizing or unlinear quantizing levels for your mode?

Mr. Kirk: In what we've done here in the paper, we assume that everything was linear. This obviously--there's a reason for doing this--if you use linear quantizing now you don't have to argue with somebody when you tell them it can be used for multi-channel telephone. It's obvious that with sync pulses sticking up on top of the video, a great number of quantizing levels could easily be saved by just saying it's either black level or now it's sync because there's only two steps there and you need not worry about it. Are there other questions? Thank you.